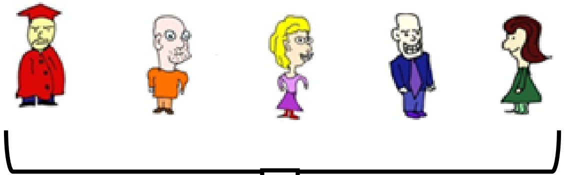
Committee Scoring Rules:
How to Choose a Good Committee?

Piotr Faliszewski
AGH University of Science and Technology
Kraków, Poland
faliszew@agh.edu.pl

Based on joint works with **Edith Elkind** (University of Oxford, UK), **Jerome Lang** (Dauphine Paris, FR), **Jean-Francois Laslier** (Paris School of Economics, FR), **Piotr Skowron** (University of Warsaw, PL), **Arkadii Slinko** (University of Auckland, NZ), **Nimrod Talmon** (Ben-Gurion University, IL)

Single-Winner Elections

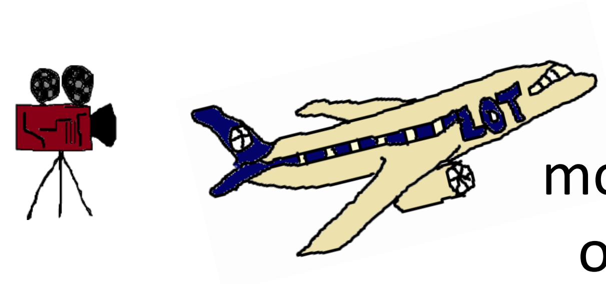


voters' preferences

voting rule

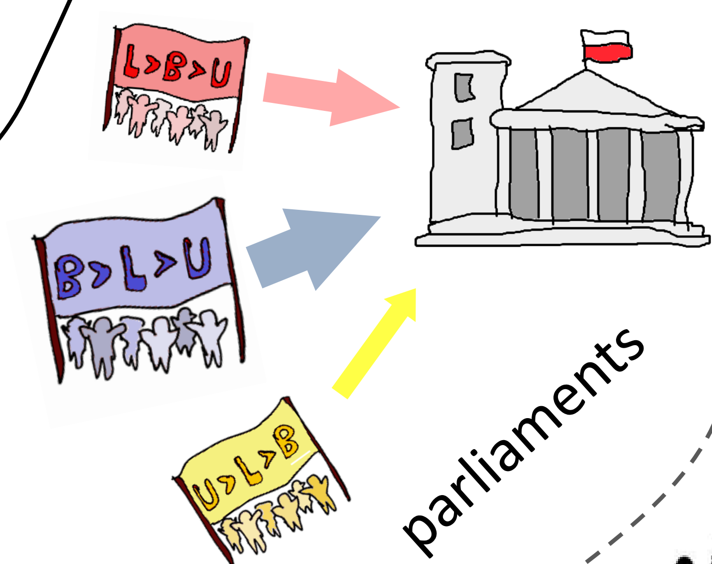
Choosing presidents, scheduling, sports/competitions
Seek the highest-ranked, most widely supported candidates

Multiwinner Elections



movies on a plane

fundamentally different!



making a shortlist

Single-Winner Scoring Rules

A single-winner scoring function:

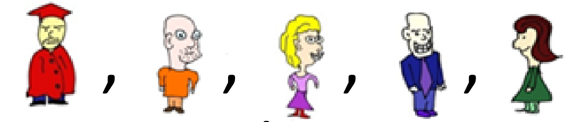
f(i) = score for position i

The candidate with the highest sum of scores is the winner

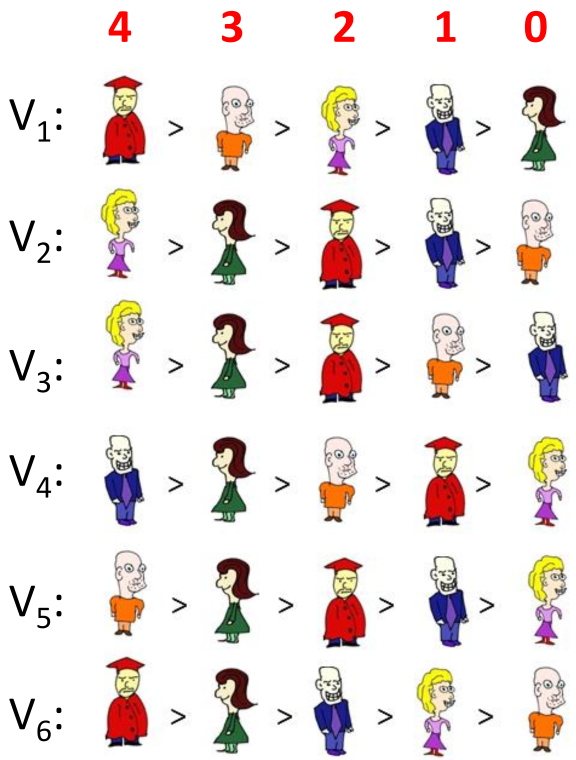
Examples:

Borda score

$\beta(i) = m-i$

C = {  }

$V = (v_1, \dots, v_6)$



Single-Winner Scoring Rules

A single-winner scoring function:

$f(i)$ = score for position i

The candidate with the highest sum of scores is the winner

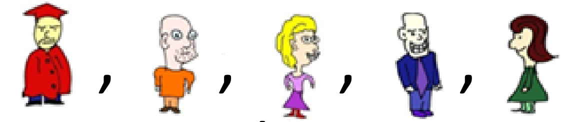
Examples:

Borda score

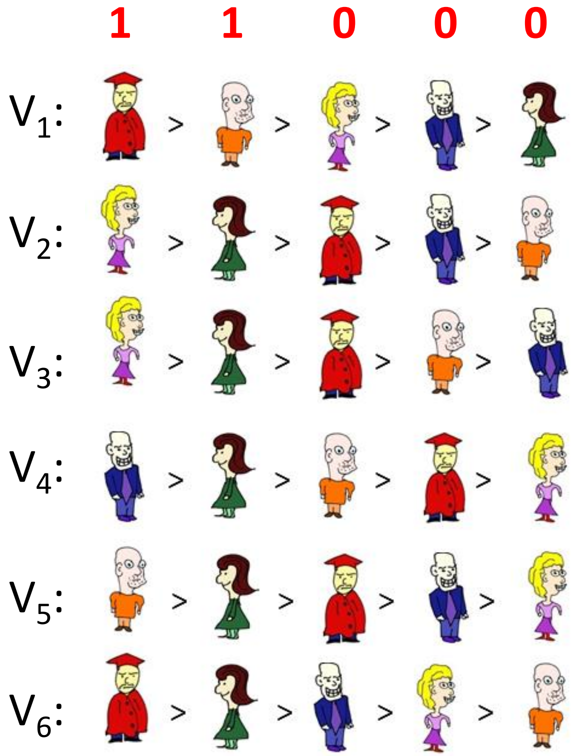
$\beta(i) = m - i$

t-Approval score

$\alpha_t(i) = 1$ if $i \leq t$ and 0 otherwise

C = {  }

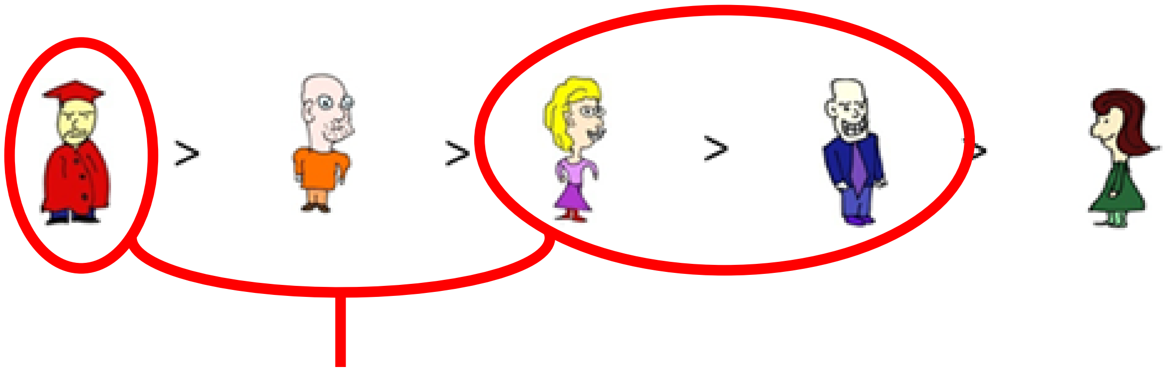
$V = (v_1, \dots, v_6)$



We Want Committee Scoring Rules

Committee Scoring Rules

Consider a preference order:



a committee

Position of the committee = (1, 3, 4)

$f(i_1, i_2, \ldots, i_k)$ = the score of the committee

Assuming $i_1 < i_2 < \ldots < i_k$

[EFSS17] E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, Properties of Multiwinner Voting Rules, Social Choice and Welfare, 2017
[SFS16] P. Skowron, P. Faliszewski, A. Slinko, Axiomatic Characterization of Committee Scoring Rules, arXiv 2016

Committee Scoring Rules

Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1) + \alpha_1(i_2) \ldots + \alpha_1(i_k)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

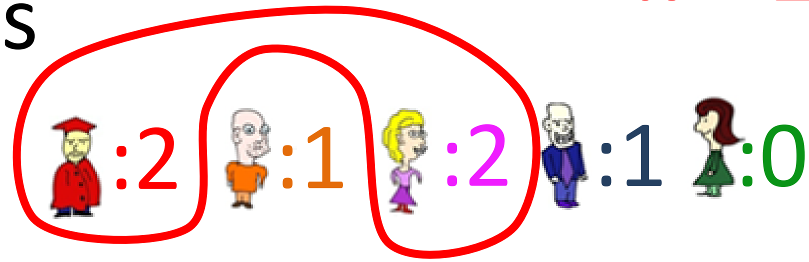
Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$

Committee Scoring Rules

k = 2



Examples

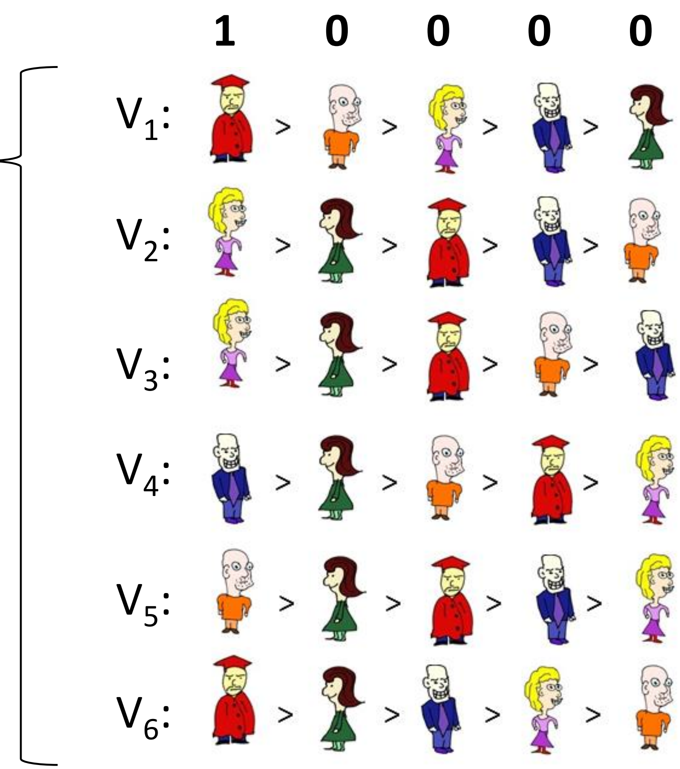
SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1) + \alpha_1(i_2) ... + \alpha_1(i_k)$

k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

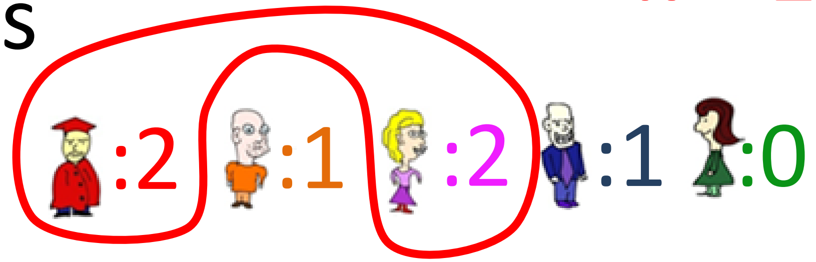
Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$



[Tul67] G. Tullock, Towards a Mathematics of Politics, Univ. of Michigan Press, 1967

Committee Scoring Rules

k = 2



Examples

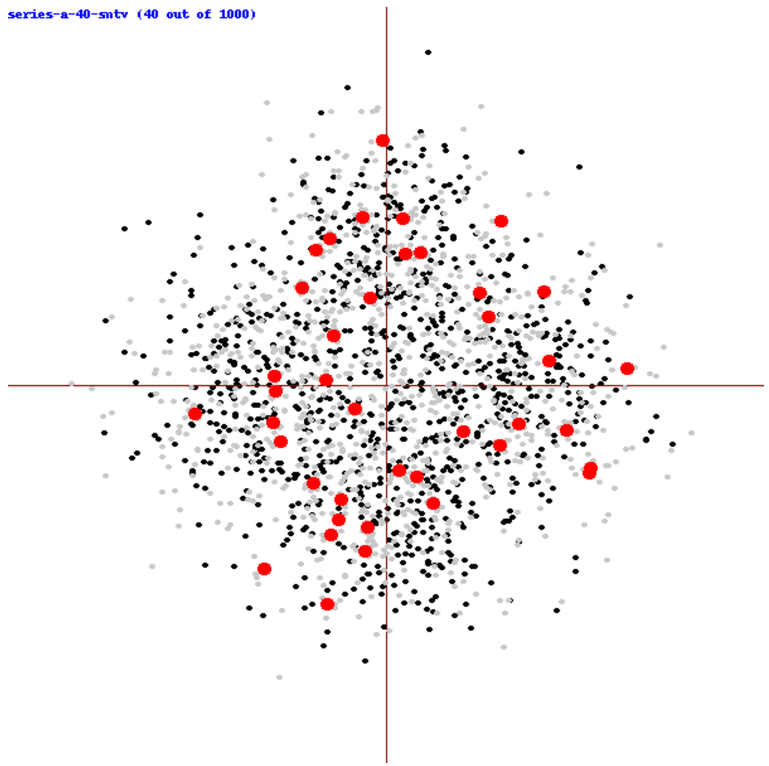
SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1) + \alpha_1(i_2) \ldots + \alpha_1(i_k)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$



[Tul67] G. Tullock, Towards a Mathematics of Politics, Univ. of Michigan Press, 1967

Committee Scoring Rules

Examples

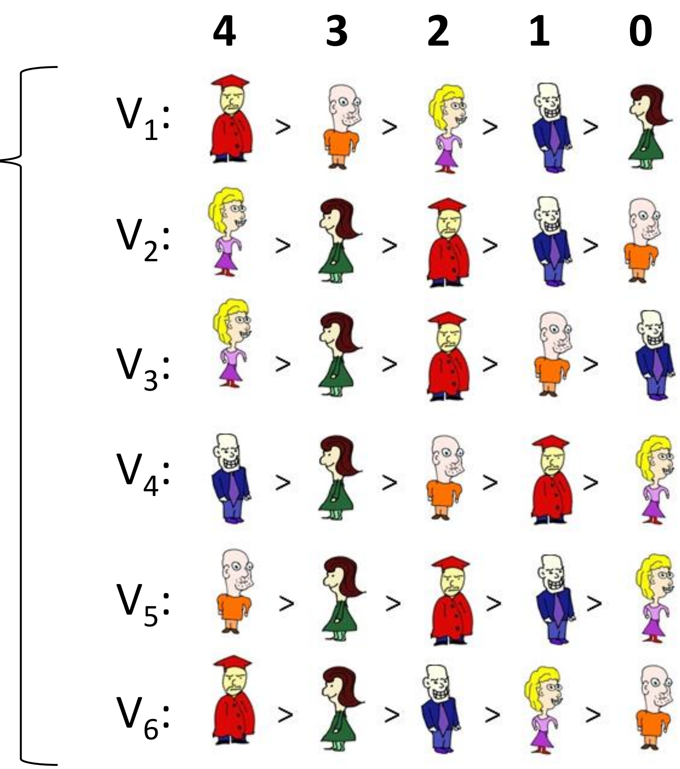
SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$\mathbf{f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)}$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$



[Deb92] B. Debord, An Axiomatic Characterization of Borda's k-Choice Function, SC&W 1992

Committee Scoring Rules

Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

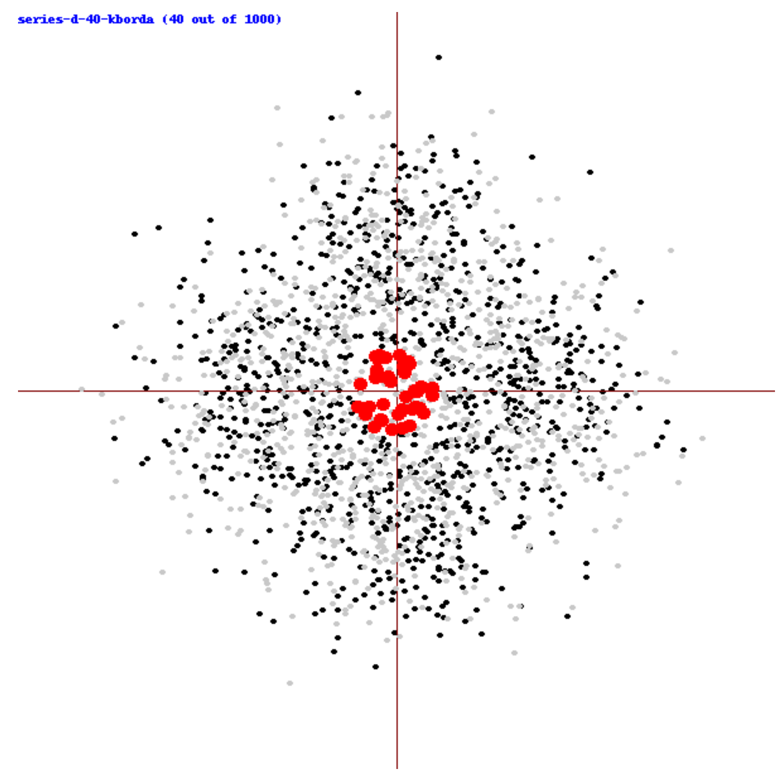
k-Borda:
$\mathbf{f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)}$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$

[Deb92] B. Debord, An Axiomatic Characterization of Borda's k-Choice Function, SC&W 1992



Committee Scoring Rules

Examples

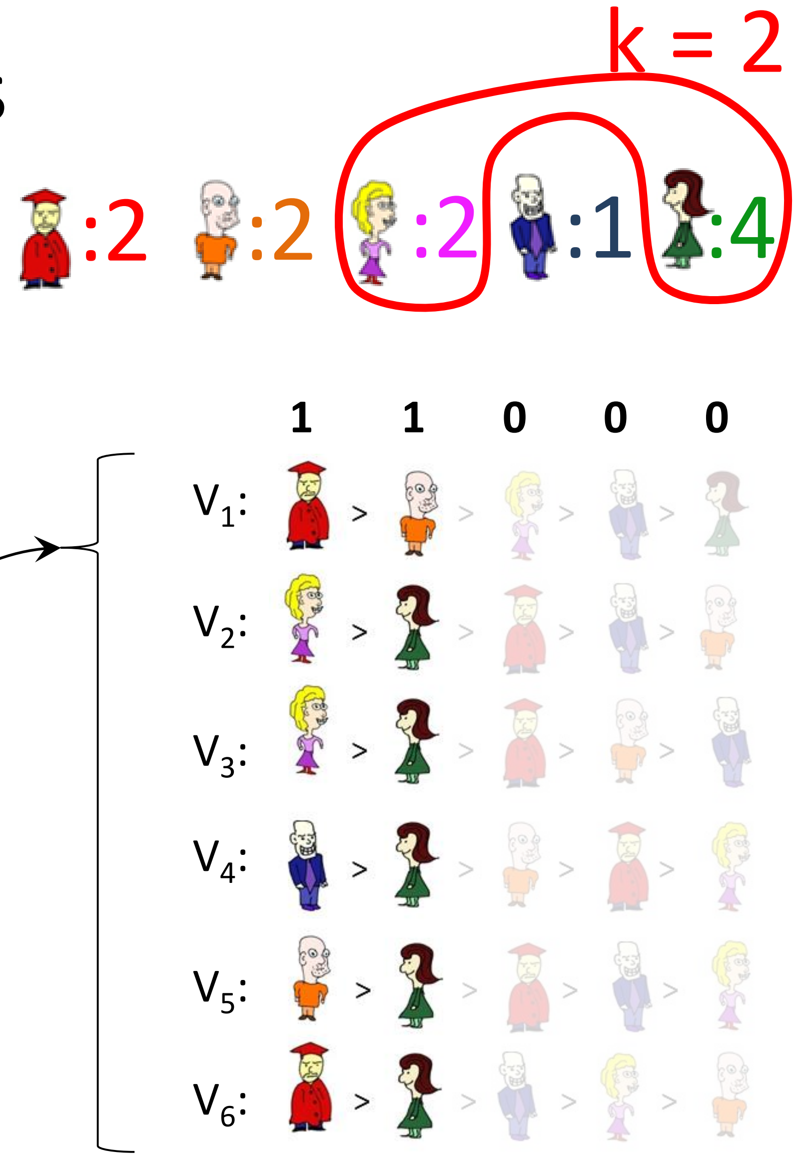
SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

Bloc:
$\mathbf{f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)}$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$



Committee Scoring Rules

Examples

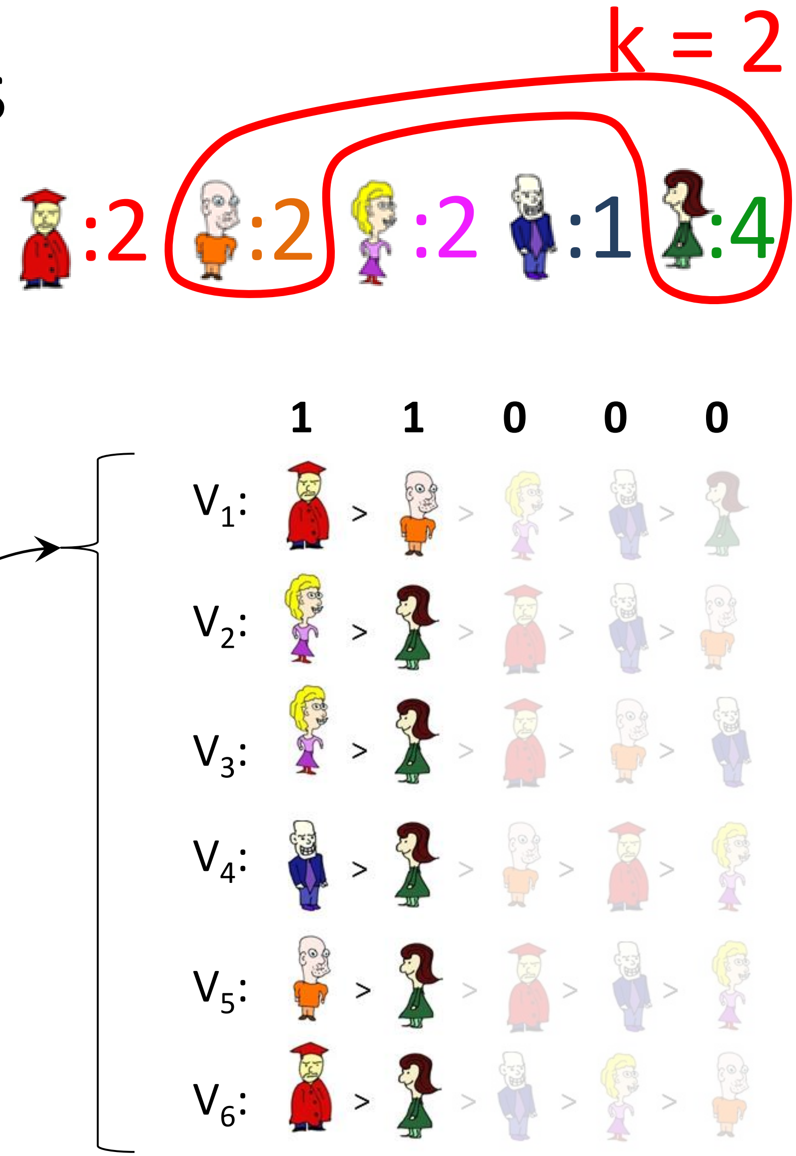
SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

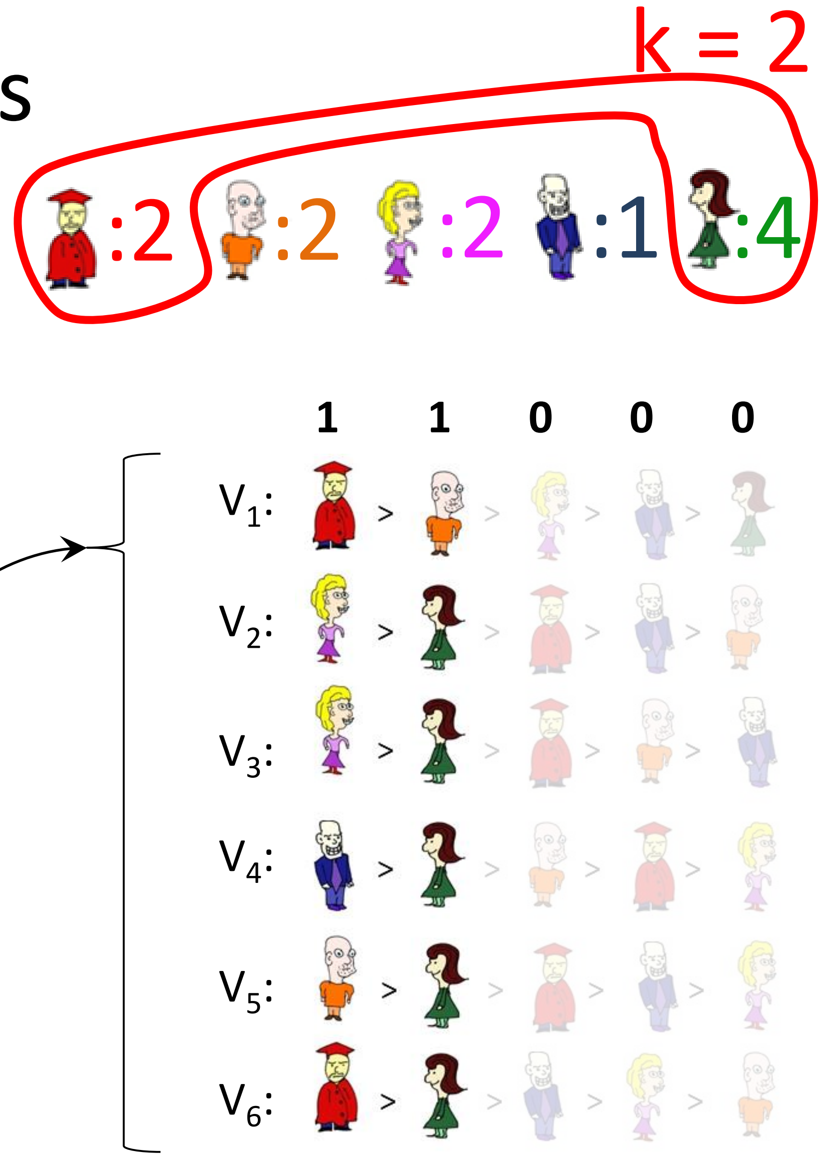
Bloc:
$\mathbf{f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)}$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$



Committee Scoring Rules



Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$\mathbf{f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)}$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

Committee Scoring Rules

Examples

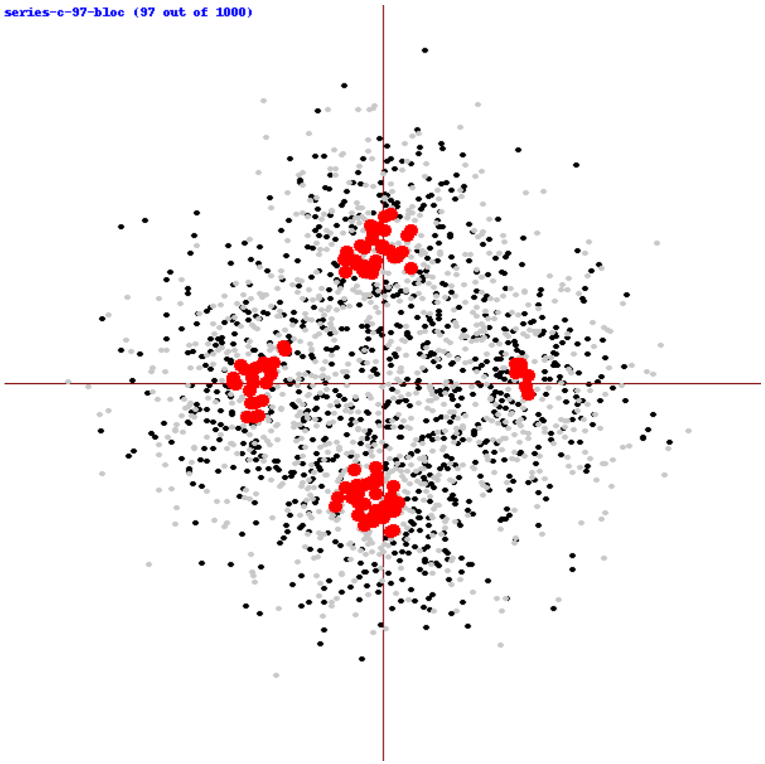
SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$\mathbf{f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)}$

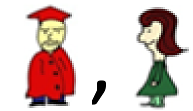
Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

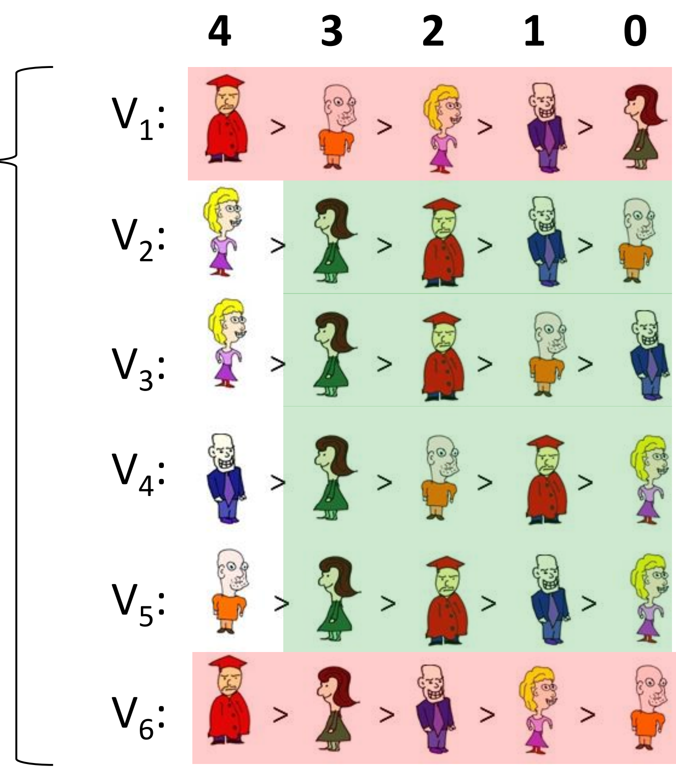


Committee Scoring Rules

k = 2

S = {  }

score(S) = 4+3+3+3+3+4 = 20



Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots + \beta(i_k)$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$\mathbf{f(i_1, \ldots, i_k) = \beta(i_1)}$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\, \alpha_k(i_k)$

[CC83] B. Chamberlin, P. Courant, Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule, Am. Pol. Sci. Rev. 1983.

Committee Scoring Rules

Examples

SNTV:
$f(i_1, \dots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \dots, i_k) = \beta(i_1) + \beta(i_2) + \dots + \beta(i_k)$

Bloc:
$f(i_1, \dots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \dots + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$\mathbf{f(i_1, \dots, i_k) = \beta(i_1)}$

Proportional Approval Voting (as CSR):
$f(i_1, \dots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \dots + 1/k\ \alpha_k(i_k)$

[CC83] B. Chamberlin, P. Courant, Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule, Am. Pol. Sci. Rev. 1983.

Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$\mathbf{f(i_1, ..., i_k) = \beta(i_1)}$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[CC83] B. Chamberlin, P. Courant, Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule, Am. Pol. Sci. Rev. 1983.

Committee Scoring Rules

Examples

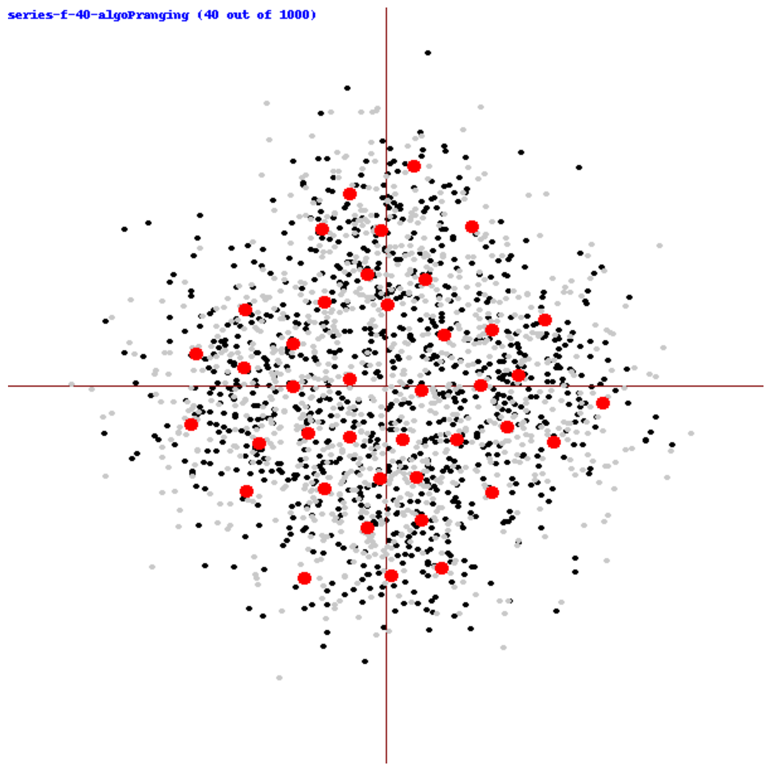
SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots + \beta(i_k)$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$



[CC83] B. Chamberlin, P. Courant, Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule, Am. Pol. Sci. Rev. 1983.

Committee Scoring Rules

Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$\mathbf{f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)}$

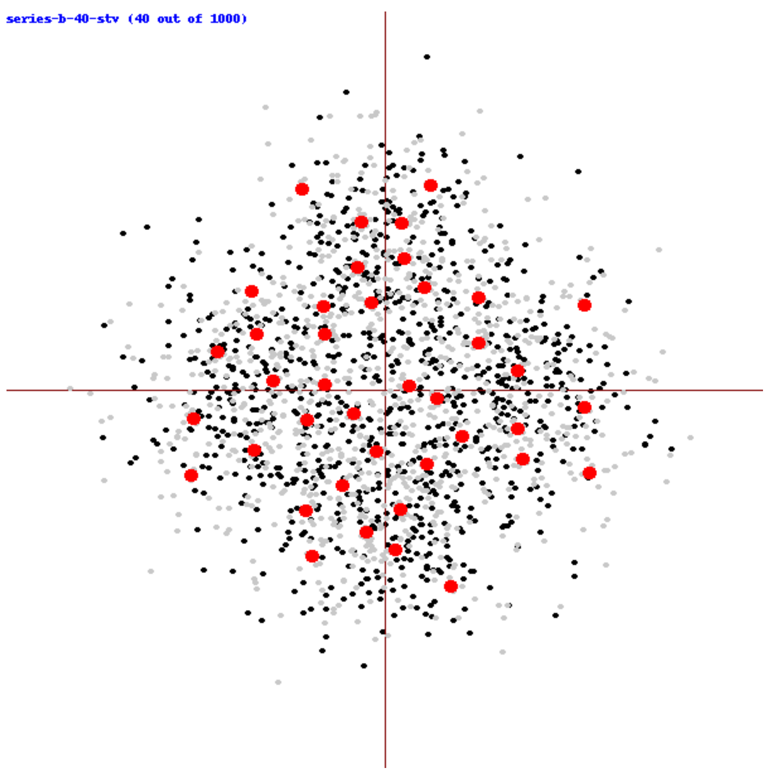
PAV: A multiwinner voting that generalizes D'Hondt apportionment method beyond party lists

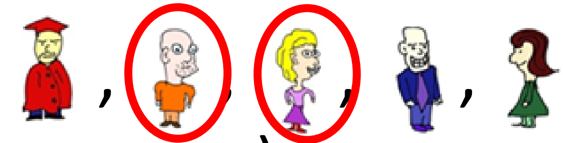
(D'Hondt method used for choosing parliaments, e.g., in France and Poland)

[BLS17] M. Brill, J-F. Laslier, P. Skowron, Multiwinner Approval Rules as Apportionment Methods, AAAI-2017.

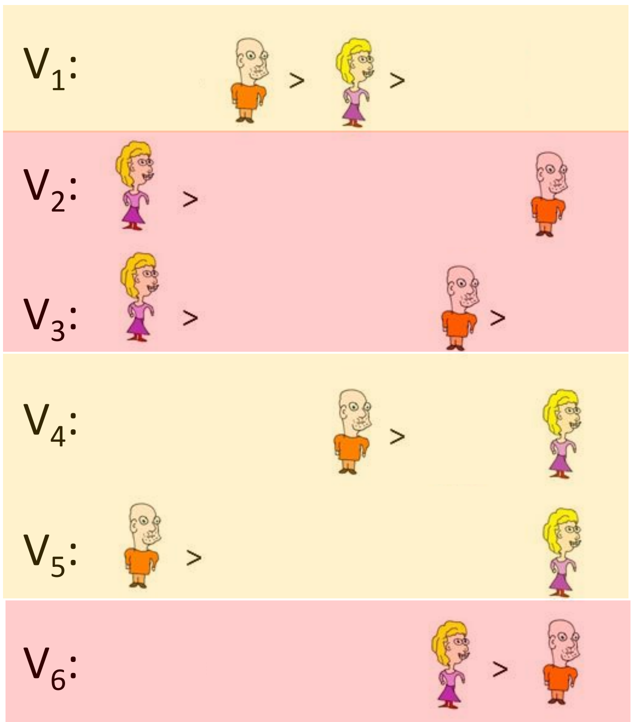
Single Transferable Vote

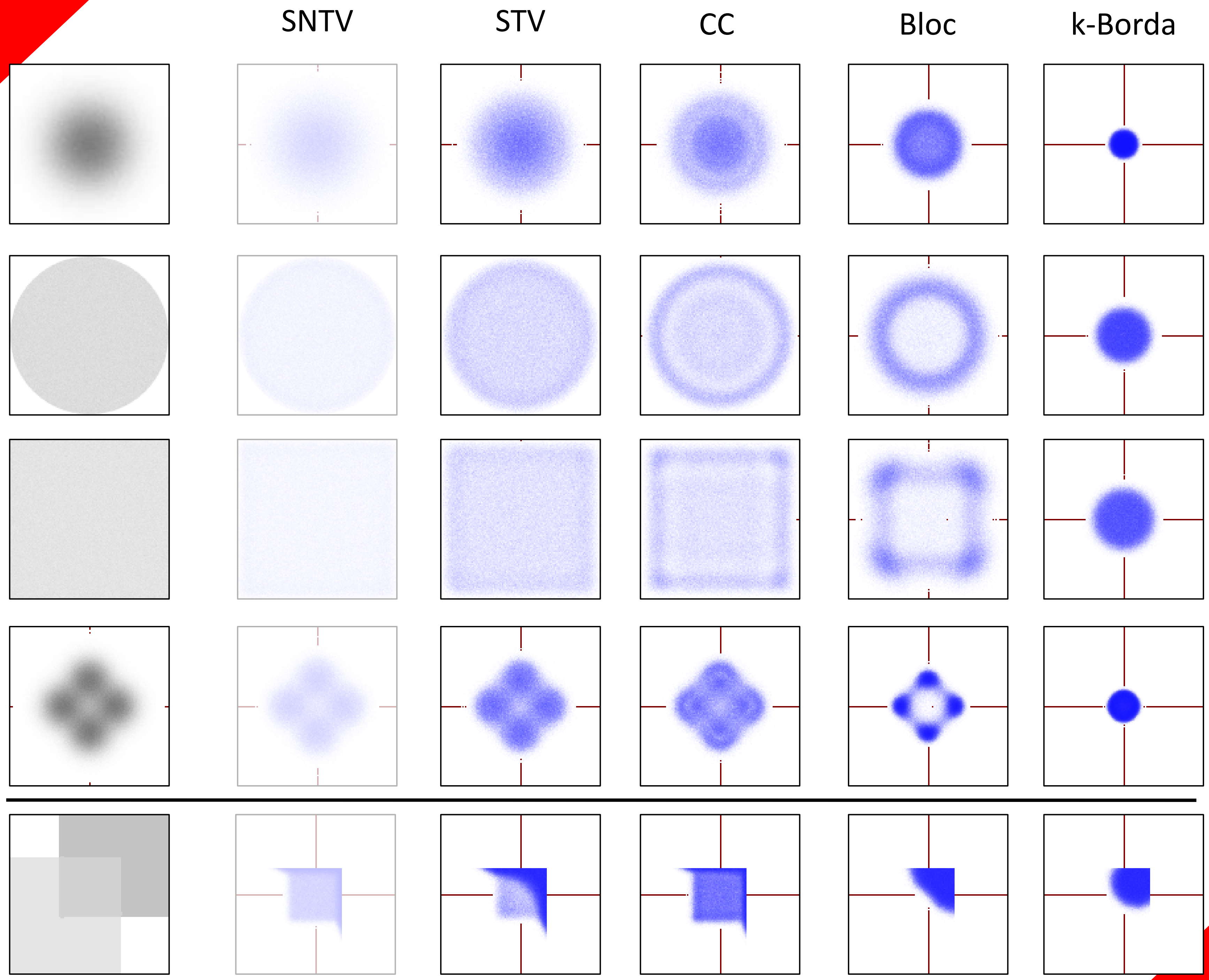
STV: Elimination process based on plurality scores (eliminate lowest scores; add to committee after reaching over n/(k+1) points)



C = {  }

$V = (v_1, \dots, v_6)$




SNTV
STV
CC
Bloc
k-Borda

We want
to understand
Committee
Scoring Rules

Committee Scoring Rules

Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

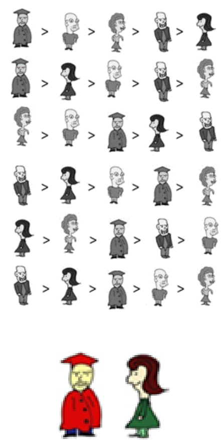
Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$

[EFSS17] E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, Properties of Multiwinner Voting Rules, Social Choice and Welfare, 2017

Consistency
If *W* is a winning committee under two elections, E_1 and E_2, then *W* is a winning committee under E_1+E_2 (and only such committees win in E_1+E_2)



Committee Scoring Rules

Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots + \beta(i_k)$

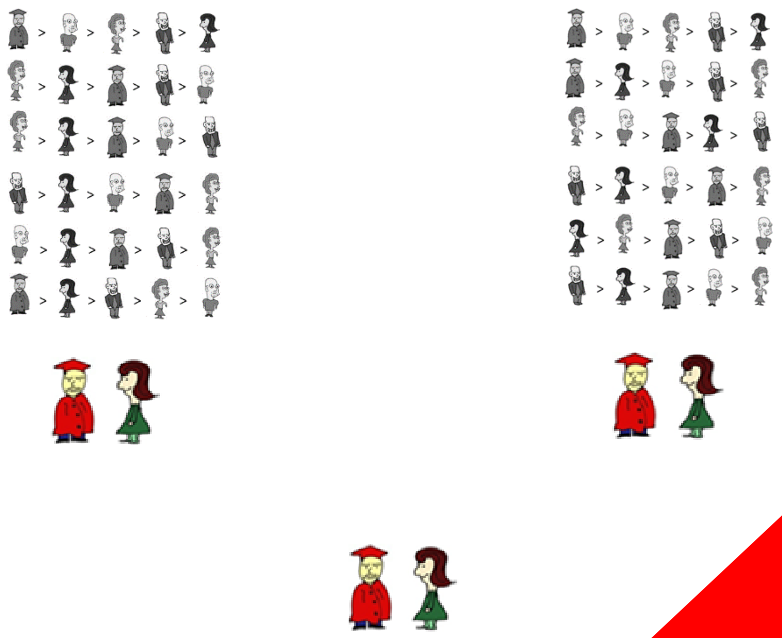
Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$

[EFSS17] E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, Properties of Multiwinner Voting Rules, Social Choice and Welfare, 2017

Consistency
If W is a winning committee under two elections, E_1 and E_2, then W is a winning committee under E_1+E_2 (and only such committees win in E_1+E_2)



Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[SFS16] P. Skowron, P. Faliszewski, A. Slinko, Axiomatic Characterization of Committee Scoring Rules, arXiv 2016.

Consistency

If *W* is a winning committee under two elections, E_1 and E_2, then *W* is a winning committee under E_1+E_2 (and only such committees win in E_1+E_2)

Theorem

Committee scoring rules are exactly the rules that satisfy consistency (+few more axioms)

Candidate Monotonicity

If a member of a winning committee *W* is shifted forward in some vote, this candidate will still belong to some winning committee (but maybe not *W*)

Theorem

All committee scoring rules satsify candidate monotonicity

Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

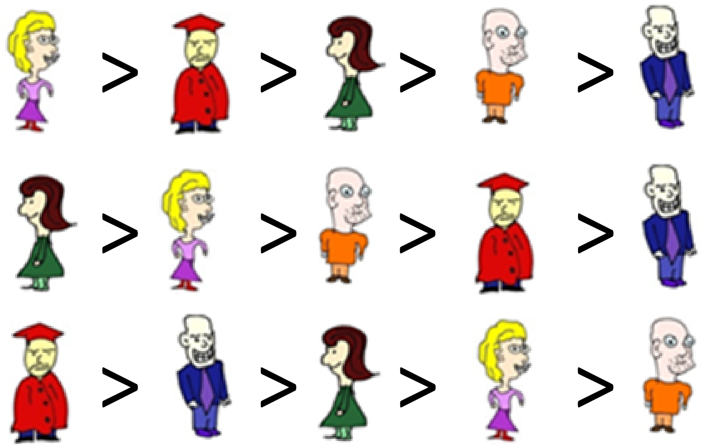
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[EFSS17] E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, Properties of Multiwinner Voting Rules, Social Choice and Welfare, 2017



Candidate Monotonicity
If a member of a winning committee *W* is shifted forward in some vote, this candidate will still belong to some winning committee (but maybe not *W*)

Theorem
All committee scoring rules satsify candidate monotonicity

Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

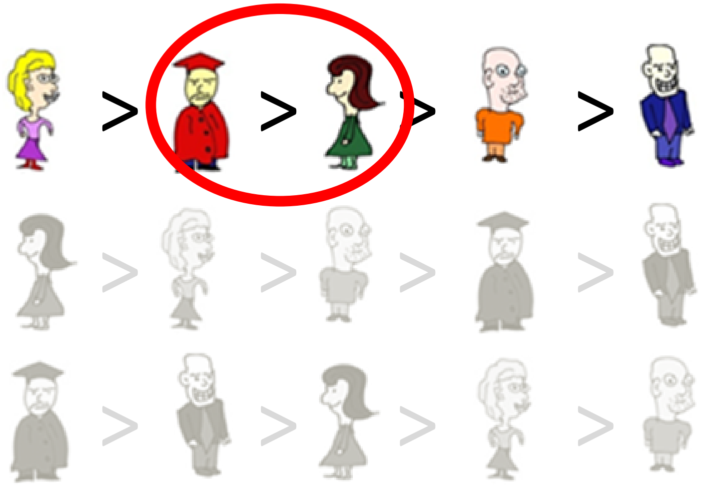
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[EFSS17] E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, Properties of Multiwinner Voting Rules, Social Choice and Welfare, 2017



Candidate Monotonicity
If a member of a winning committee *W* is shifted forward in some vote, this candidate will still belong to some winning committee (but maybe not *W*)

Theorem
All committee scoring rules satsify candidate monotonicity

Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

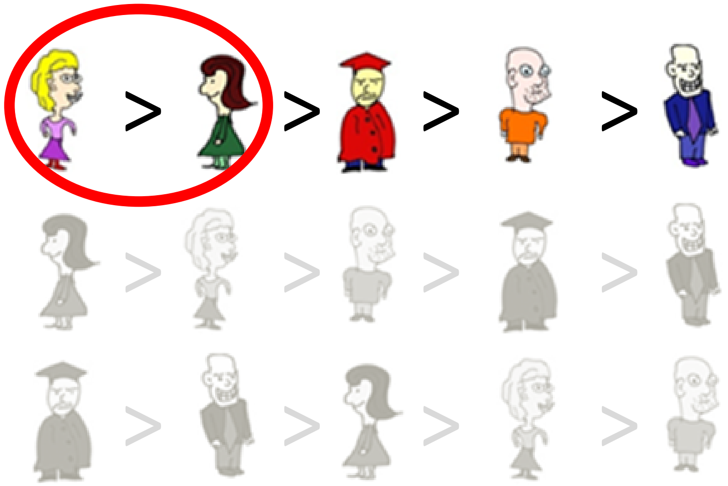
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[EFSS17] E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, Properties of Multiwinner Voting Rules, Social Choice and Welfare, 2017



Candidate Monotonicity
If a member of a winning committee *W* is shifted forward in some vote, this candidate will still belong to some winning committee (but maybe not *W*)

Theorem
All committee scoring rules satsify candidate monotonicity

Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Committee Scoring Rules

Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1) + \alpha_1(i_2) + ... + \alpha_1(i_k)$

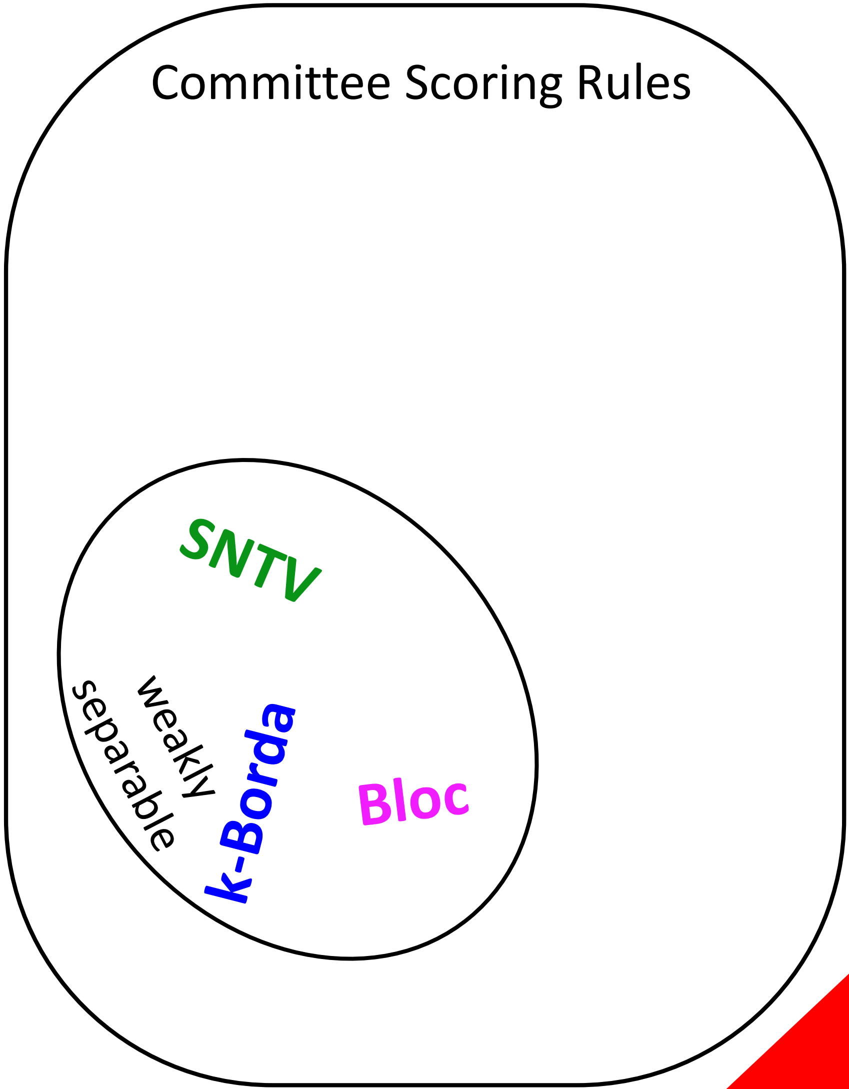
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016



Committee Scoring Rules

Examples

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

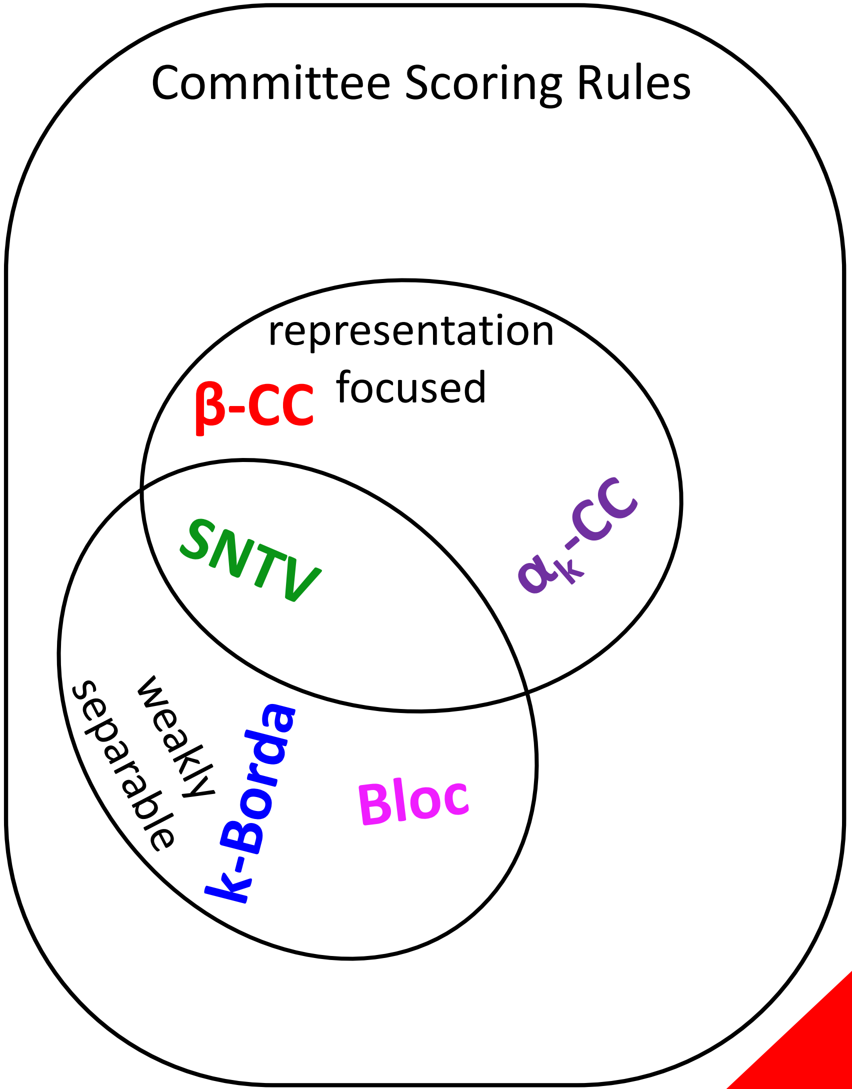
k-Borda:
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

Bloc:
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, \ldots, i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, \ldots, i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + \ldots + 1/k\ \alpha_k(i_k)$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016



Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

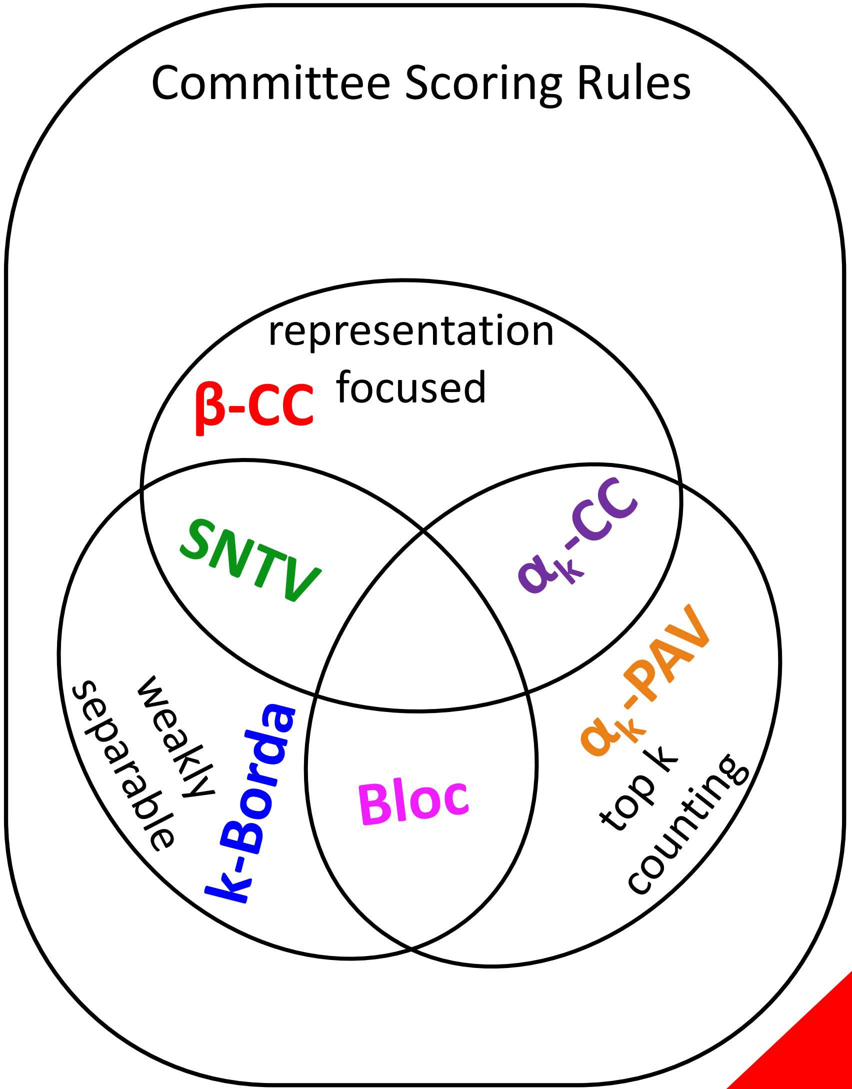
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$\mathbf{f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)}$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$\mathbf{f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\, \alpha_k(i_k)}$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016



Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

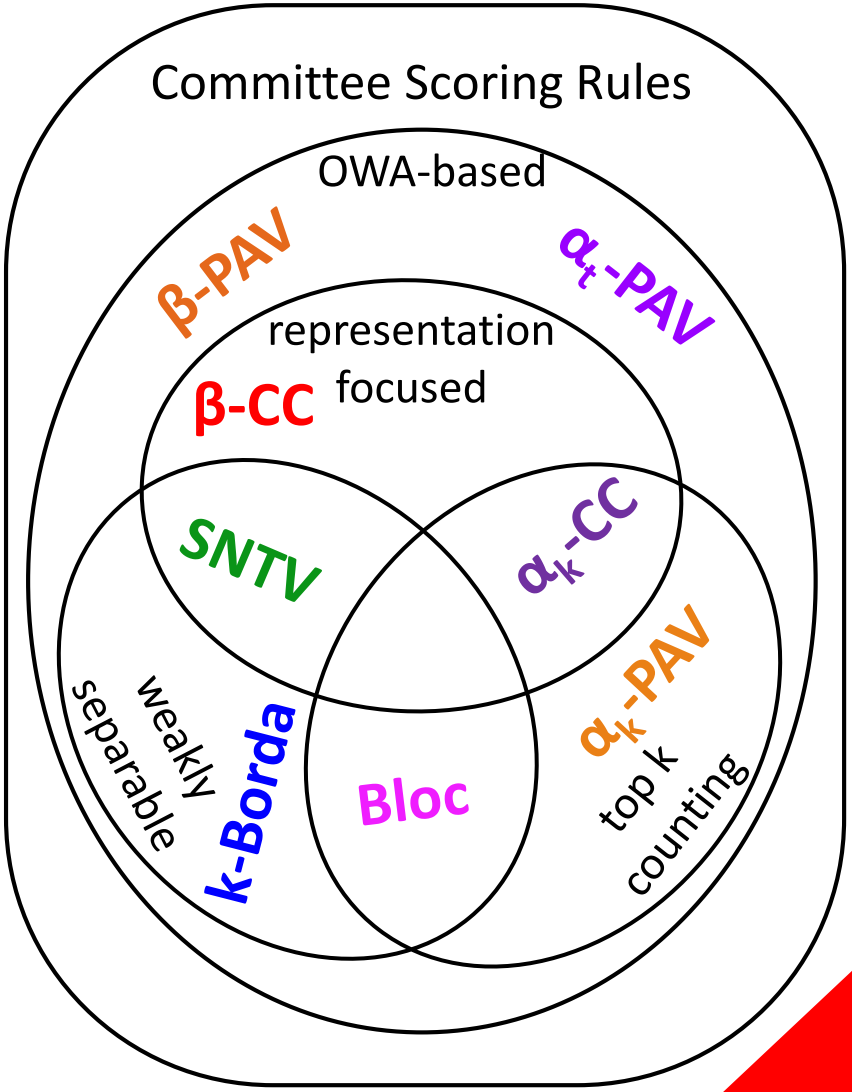
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016



Committee Scoring Rules

Examples

SNTV:
$f(i_1, ..., i_k) = \alpha_1(i_1)$

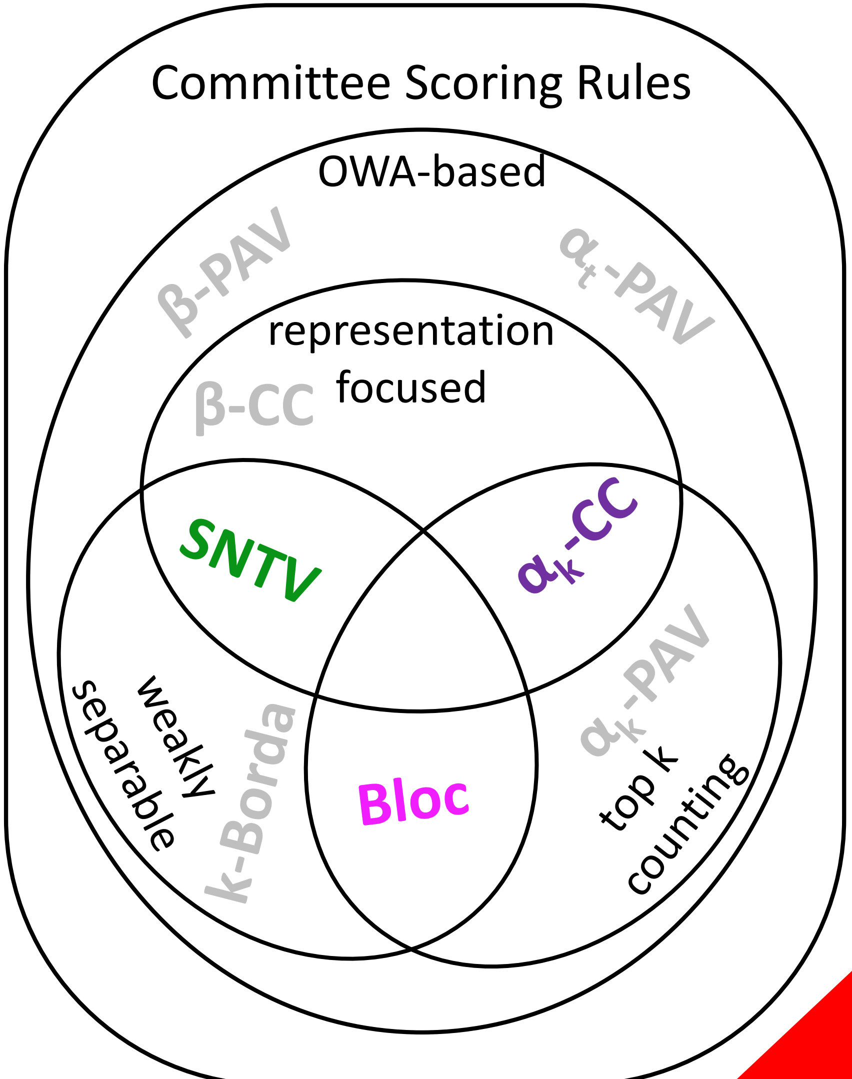
k-Borda:
$f(i_1, ..., i_k) = \beta(i_1) + \beta(i_2) + ... + \beta(i_k)$

Bloc:
$f(i_1, ..., i_k) = \alpha_k(i_1) + \alpha_k(i_2) + ... + \alpha_k(i_k)$

Chamberlin—Courant (β-CC):
$f(i_1, ..., i_k) = \beta(i_1)$

Proportional Approval Voting (as CSR):
$f(i_1, ..., i_k) = \alpha_k(i_1) + 1/2\alpha_k(i_2) + ... + 1/k\ \alpha_k(i_k)$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016



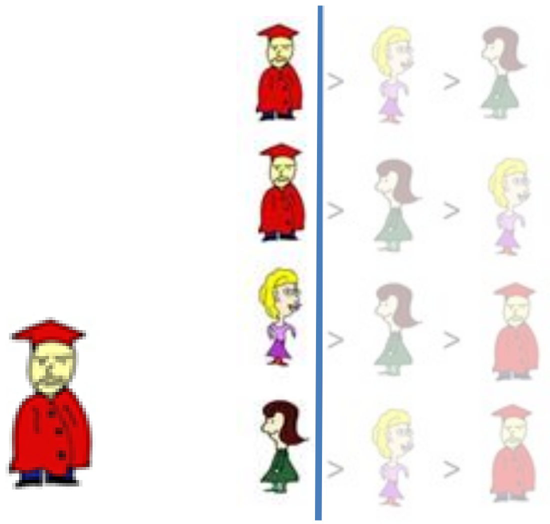
Nature of the Committees

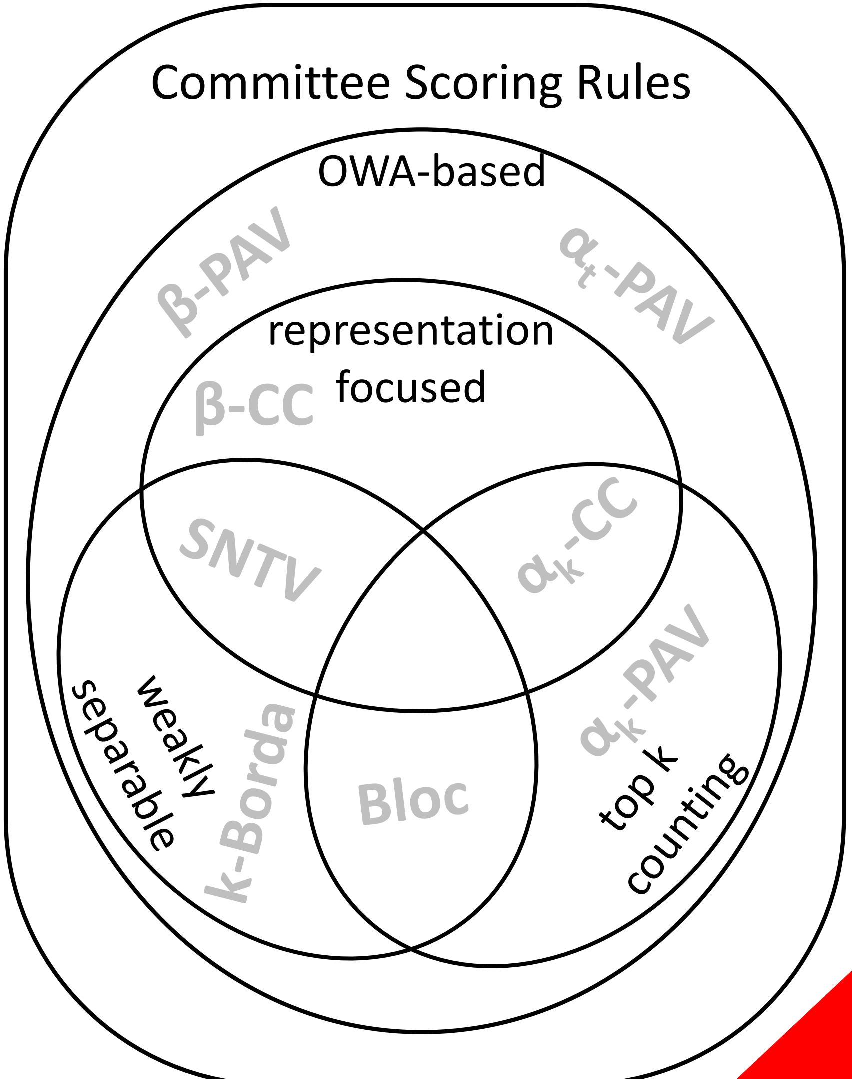
(Individual Excellence)

Committee Scoring Rules

Committee Monotonicity: If a candidate is selected for a committee of size k, then this candidate is also selected for committee of size k+1

Problem with Bloc



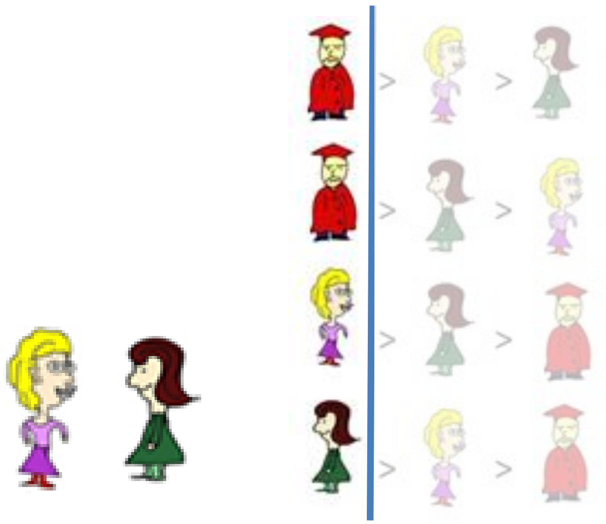


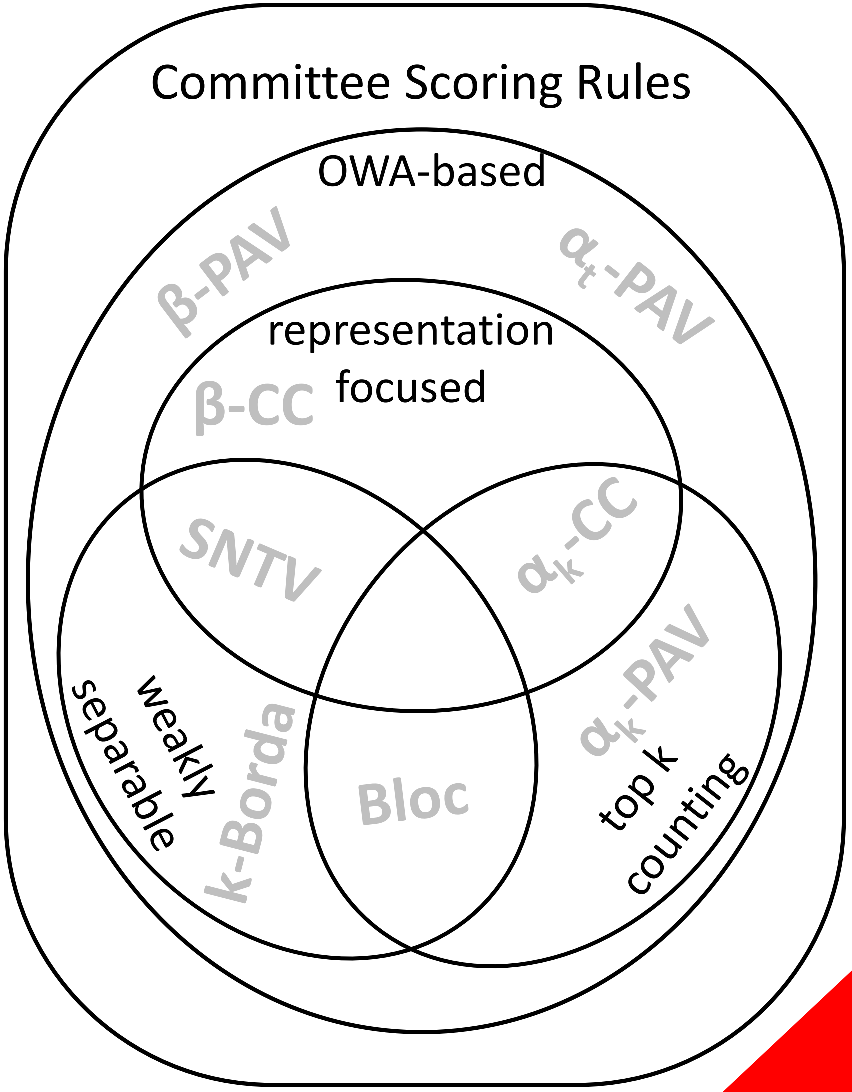
[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Committee Scoring Rules

Committee Monotonicity: If a candidate is selected for a committee of size k, then this candidate is also selected for committee of size k+1

Problem with Bloc





[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Committee Scoring Rules

Committee Monotonicity: If a candidate is selected for a committee of size k, then this candidate is also selected for committee of size k+1

Theorem A committee scoring rule is committee monotone if and only if it is separable.

Separable Rules

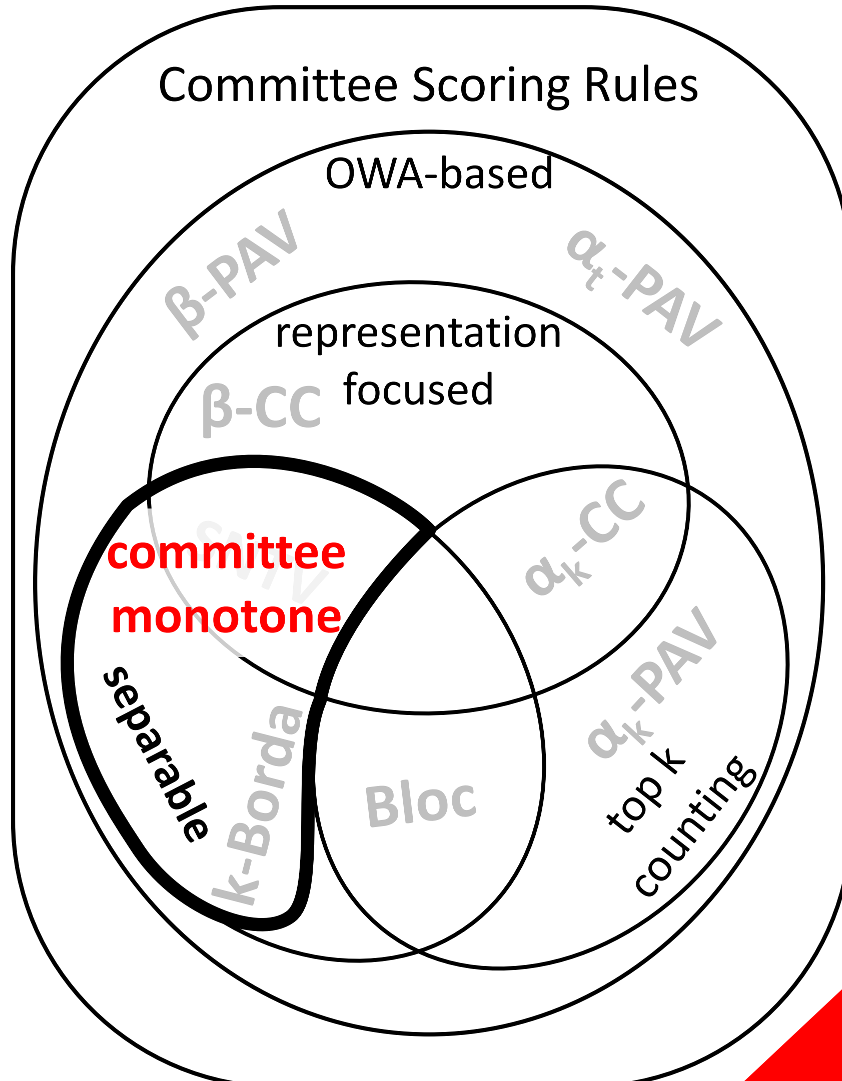
SNTV:

$f(i_1, \ldots, i_k) = \alpha_1(i_1) + \alpha_1(i_2) + \ldots + \alpha_1(i_k)$

k-Borda:

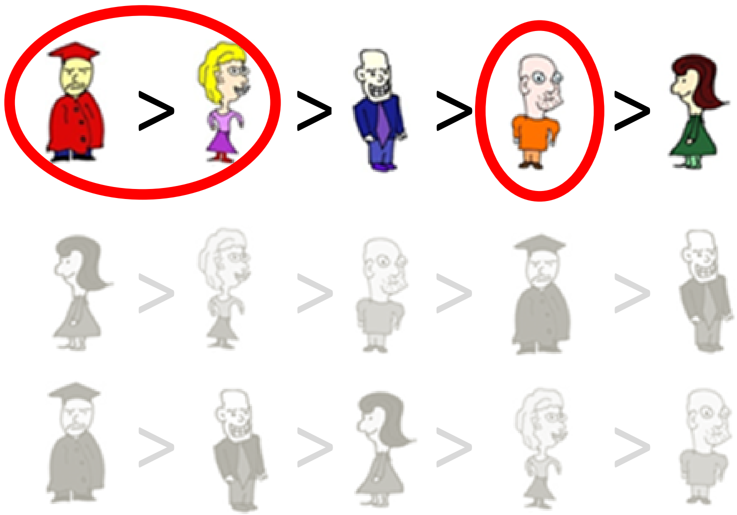
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$

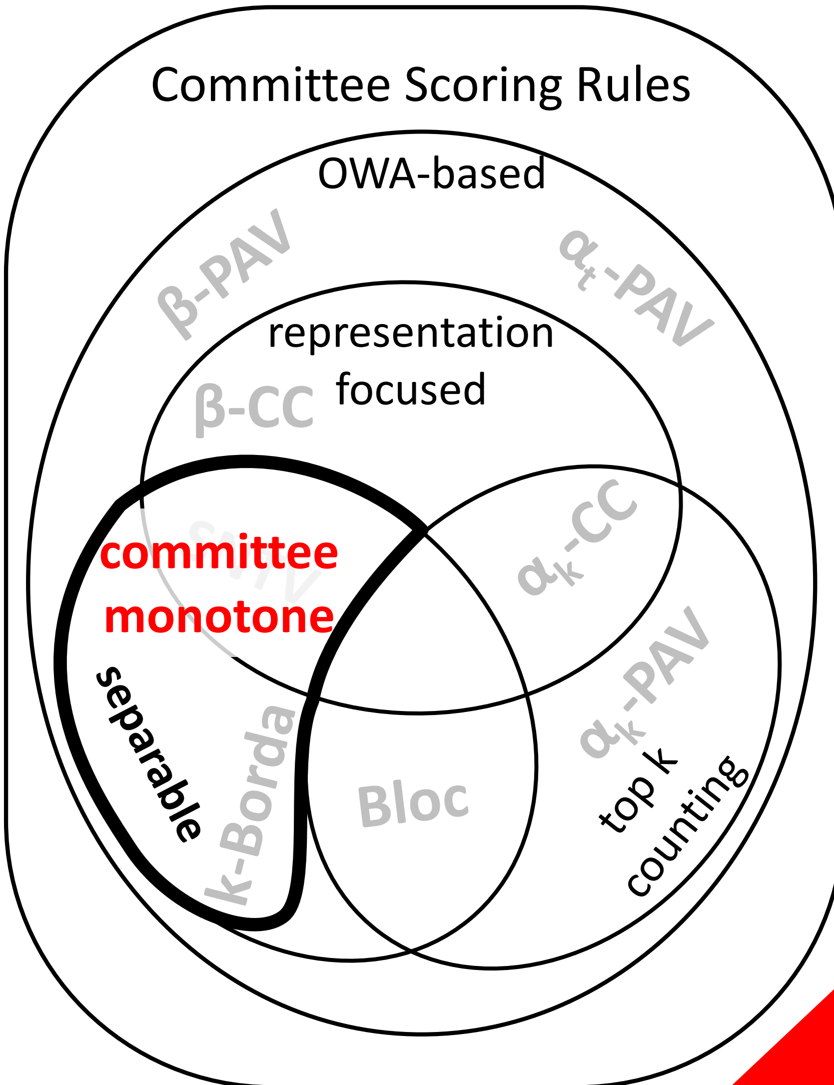
[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016



Committee Scoring Rules

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

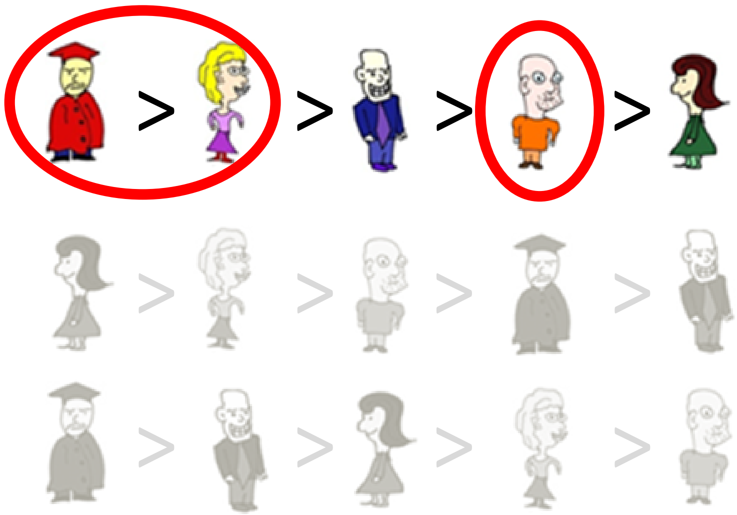


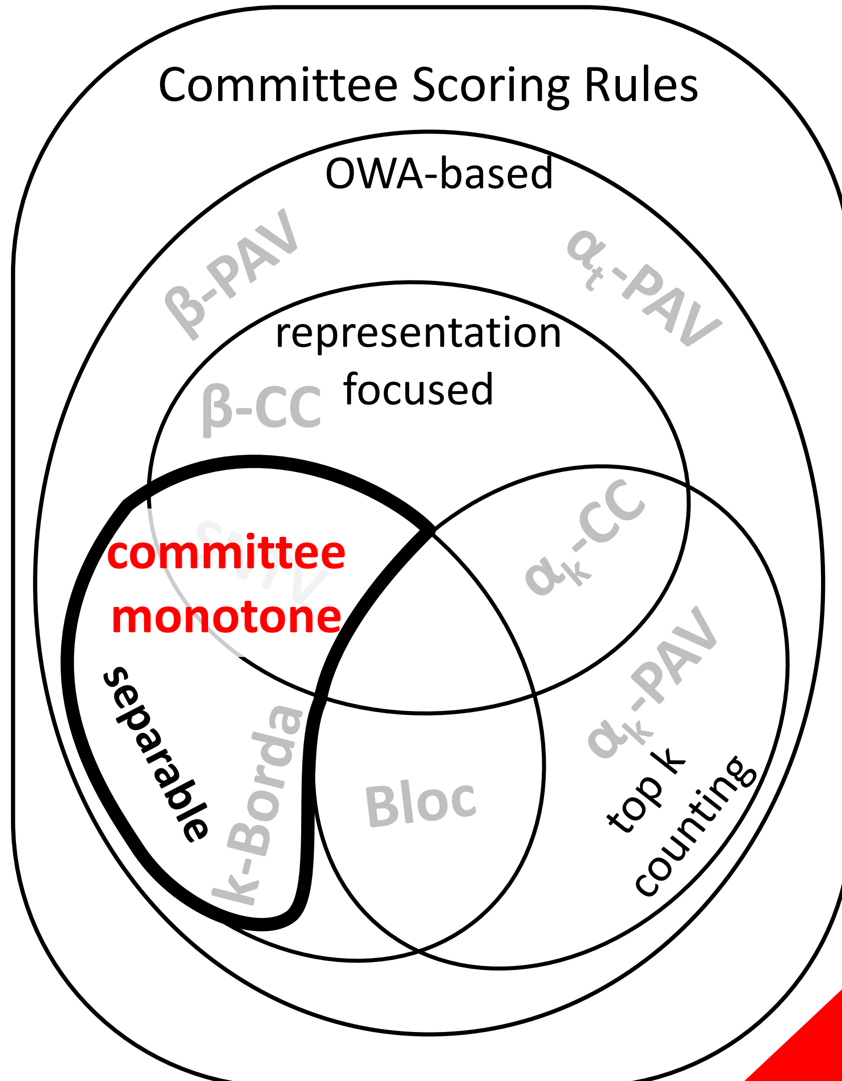


[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Committee Scoring Rules

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning





[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Committee Scoring Rules

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

Theorem A committee scoring rule is noncrossing monotone if and only if it is weakly separable.

Weakly Separable Rules

SNTV:

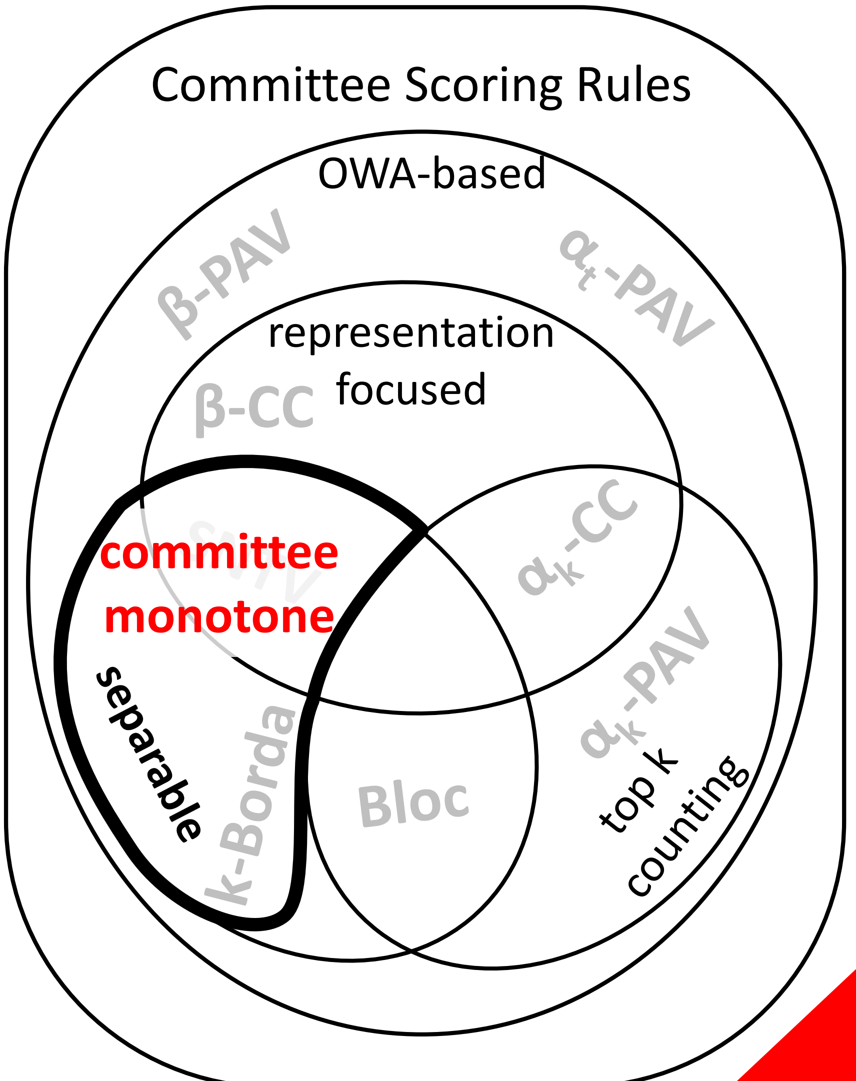
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

Bloc:

$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots. + \alpha_k(i_k)$

k-Borda:

$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots. + \beta(i_k)$



[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Committee Scoring Rules

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

Theorem A committee scoring rule is noncrossing monotone if and only if it is weakly separable.

Weakly Separable Rules

SNTV:

$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

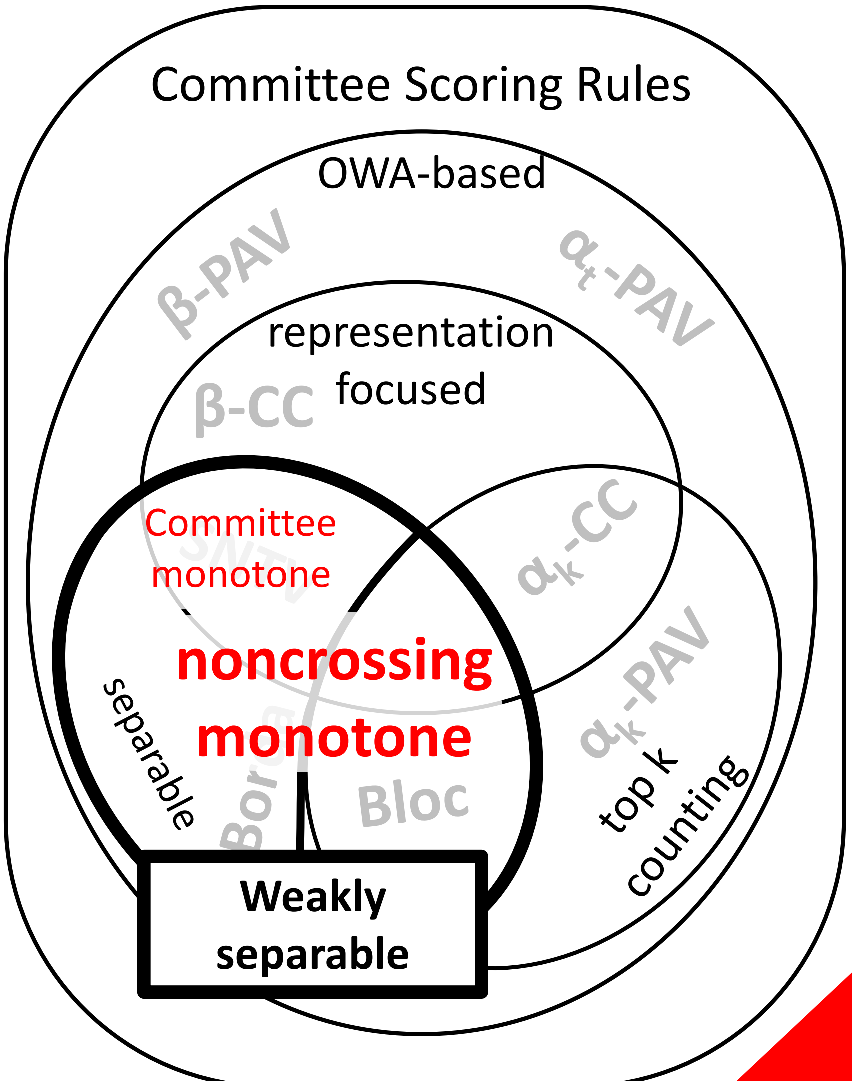
Bloc:

$f(i_1, \ldots, i_k) = \alpha_k(i_1) + \alpha_k(i_2) + \ldots + \alpha_k(i_k)$

k-Borda:

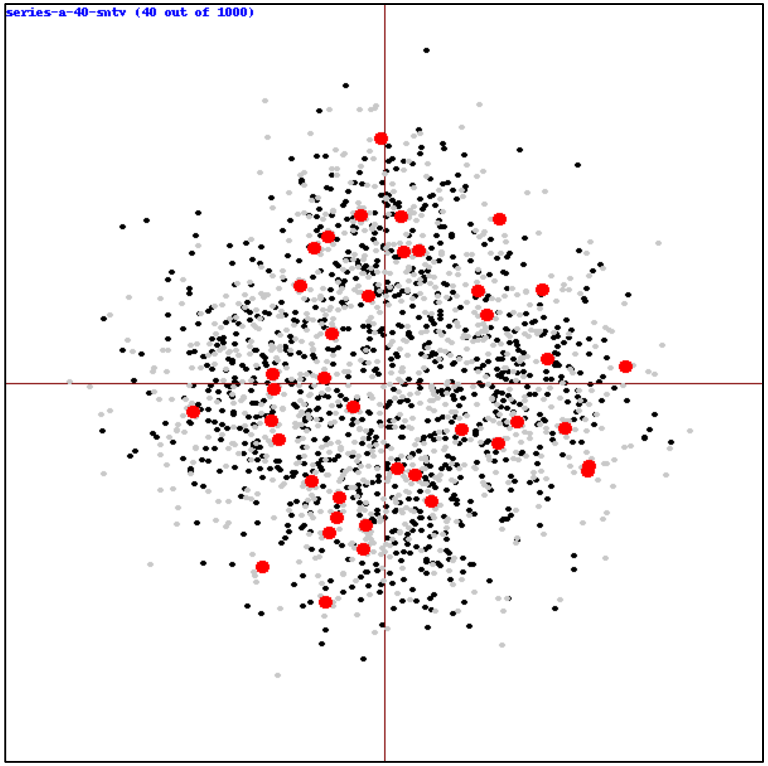
$f(i_1, \ldots, i_k) = \beta(i_1) + \beta(i_2) + \ldots + \beta(i_k)$

[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

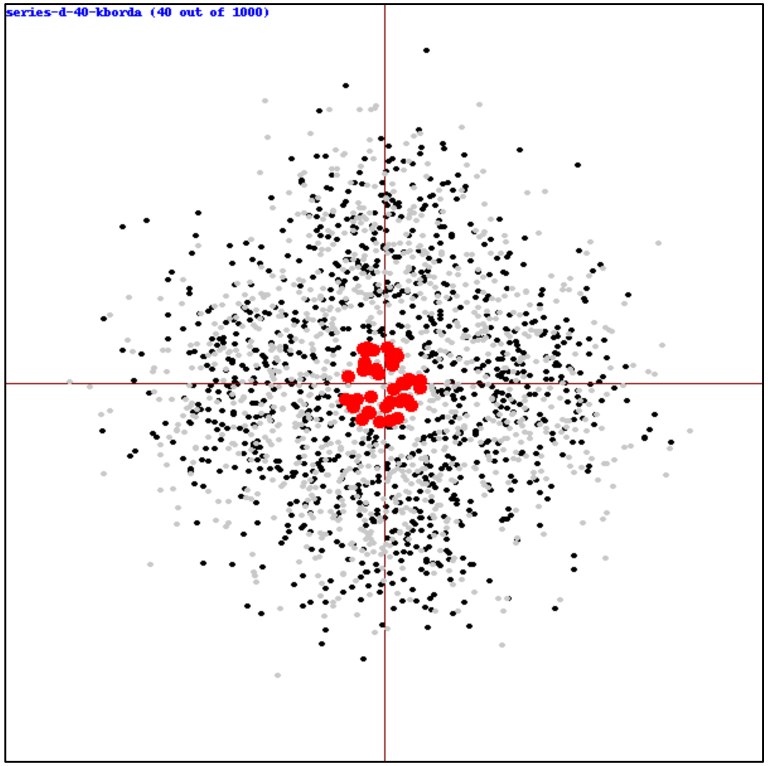


Is SNTV really good for individual excellence?

SNTV



k-Borda



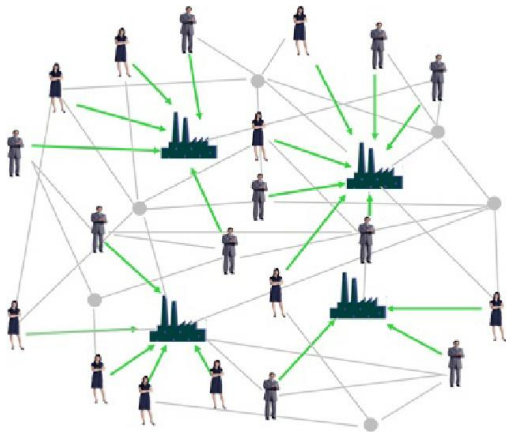
Nature of the Committees

(Diveristy/Coverage)

Applications Requiring Diversity/Coverage

Instead of finding the "best" candidates (recall Excellence) we aim at covering **all** views of the electorate

Some applications:



Where to place facilities?

Which products to produce?

Which products to advertise?

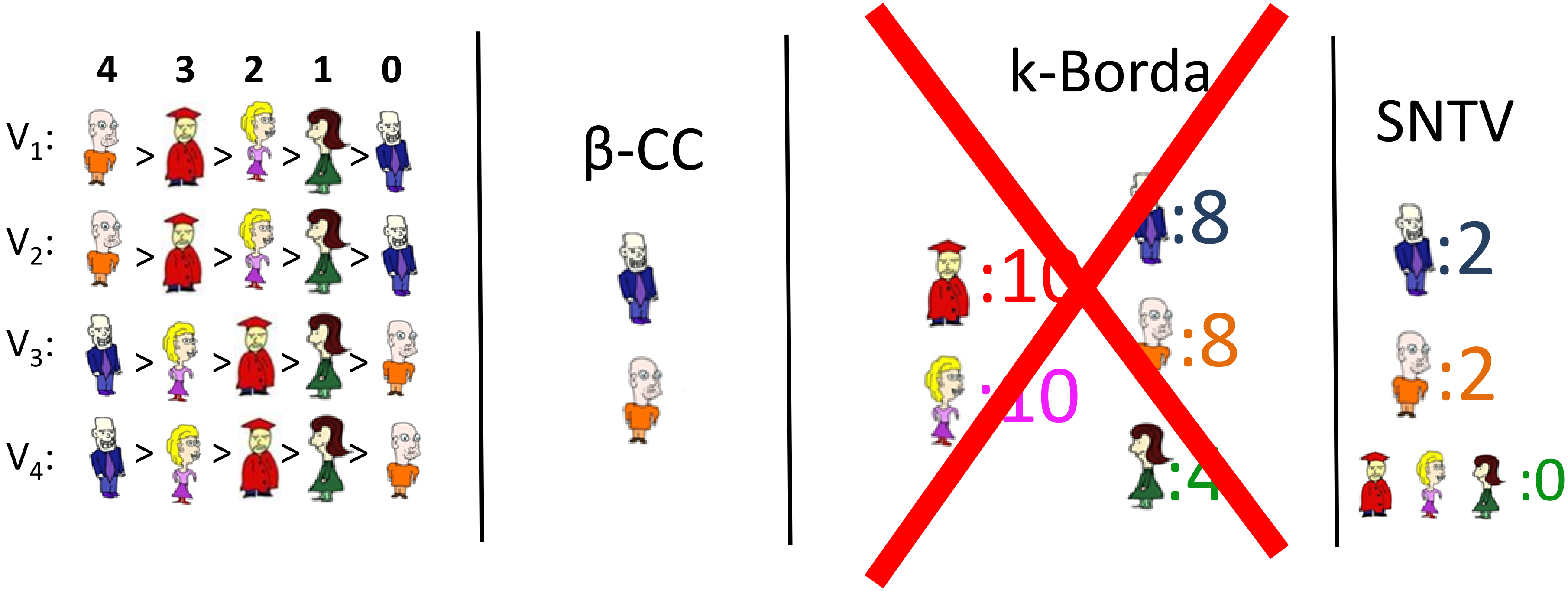
Axioms for Diversity: Narrow Top

Narrow Top

A rule satisfies the **narrow top** criterion if whenever there is a set *W* of *k* candidates such that each voter ranks first a member of W, then W is a winning committee

β-CC satisfies narrow top
k-Borda (e.g.,) does not

and SNTV



E. Elkind, P. Faliszewski, P. Skowron, A. Slinko: Properties of Multiwinner Voting Rules, SC&W, 2017

Axioms for Diversity: Narrow Top

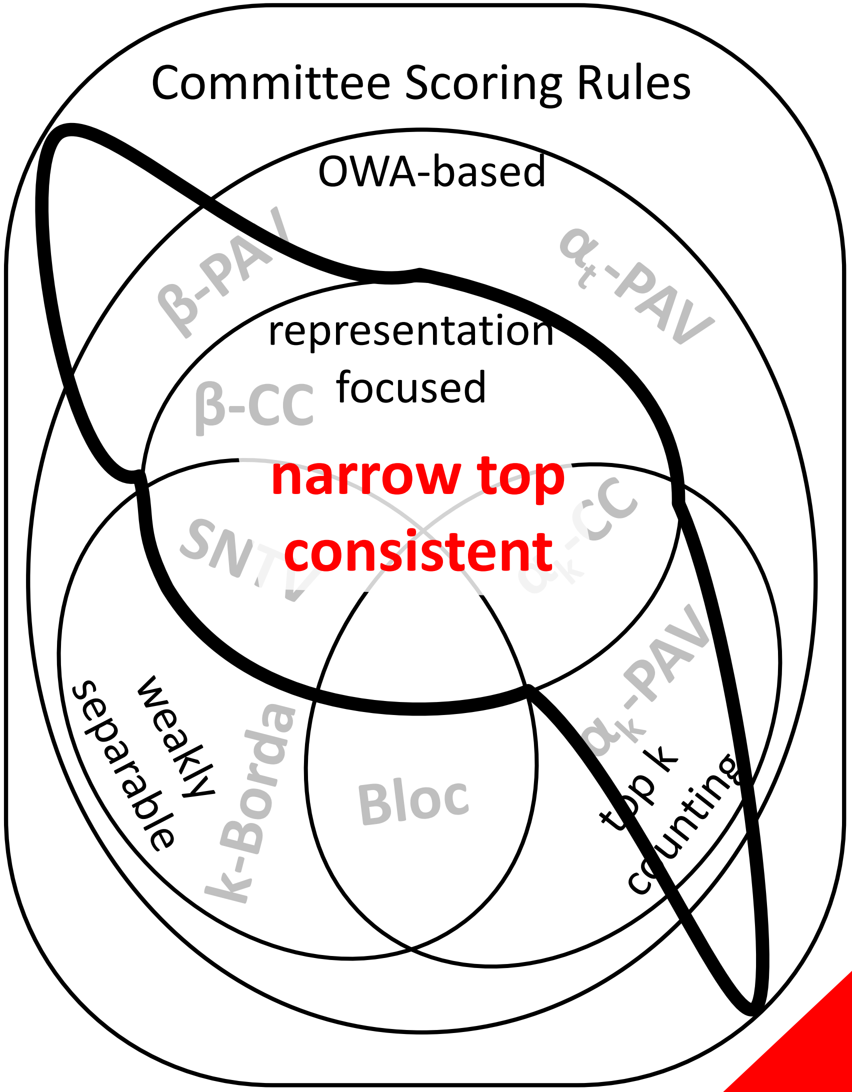
Narrow Top
A rule satisfies the **narrow top** criterion if whenever there is a set *W* of *k* candidates such that each voter ranks first a member of W, then W is a winning committee

Theorem If a committee scoring rule is representation-focused then it is narrow-top consistent.

Representation-Focused Rules

SNTV:
$f(i_1, \ldots, i_k) = \alpha_1(i_1)$

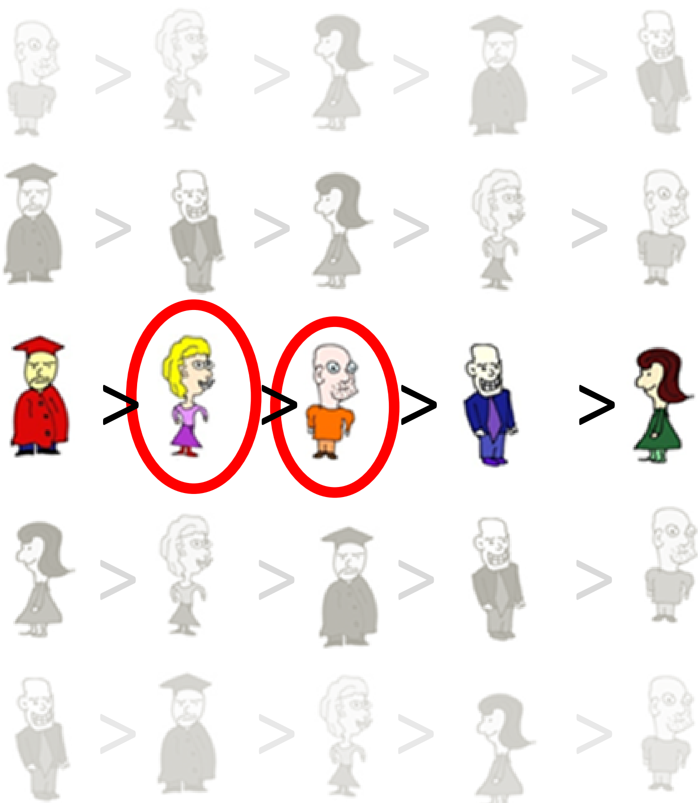
β-CC:
$f(i_1, \ldots, i_k) = \beta(i_1)$



[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Axiom: Top-member monotonicity

Top-Member Monotonicity: If the highest ranked member of the winning committee is moved forward, the committee still wins.

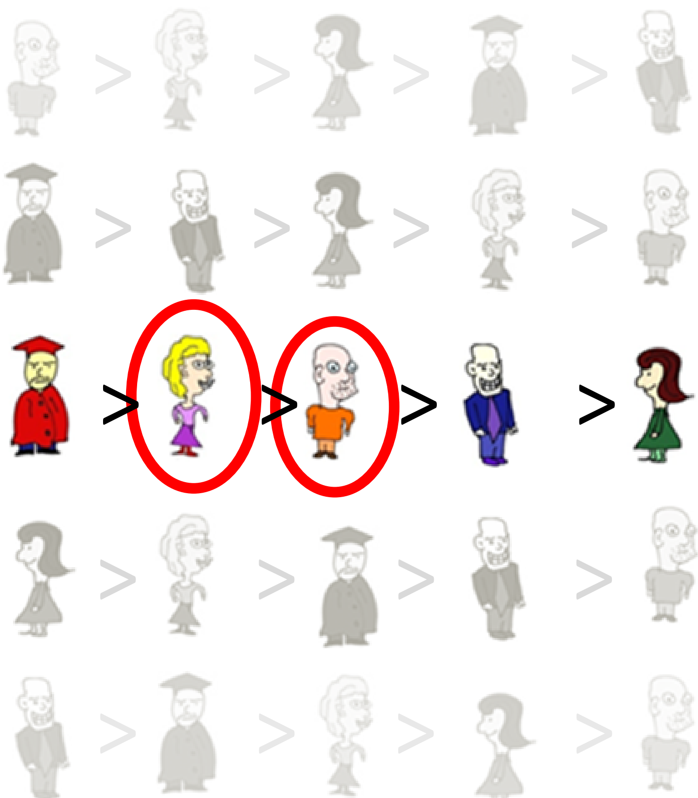


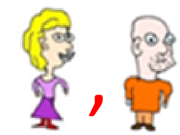
score( **,**  **) = X**

Axiom: Top-member monotonicity

Top-Member Monotonicity: If the highest ranked member of the winning committee is moved forward, the committee still wins.

β-CC satisfies top-member monotonicity

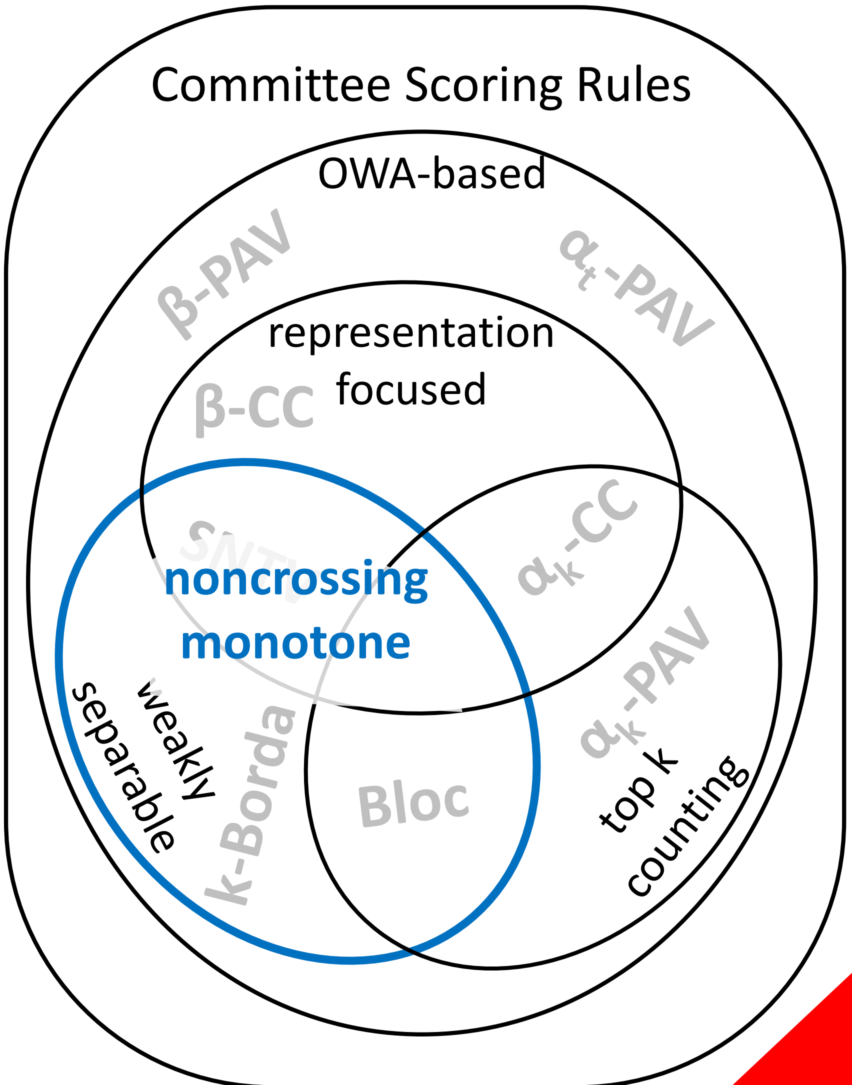


score() = X+1

The shift gives the same number of points to every committee where the candidate is top member

Axiom: Top-member monotonicity

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

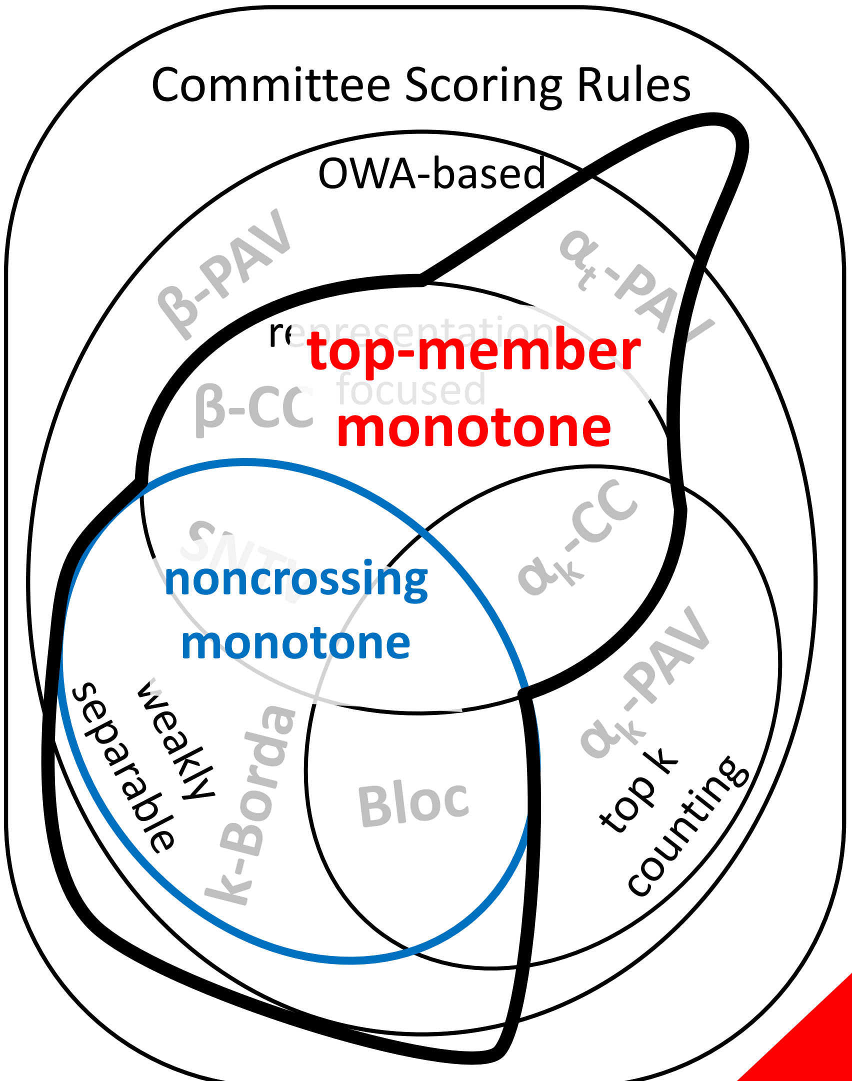


[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Axiom: Top-member monotonicity

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

Top-Member Monotonicity: If the highest ranked member of the winning committee is moved forward, the committee still wins.



[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

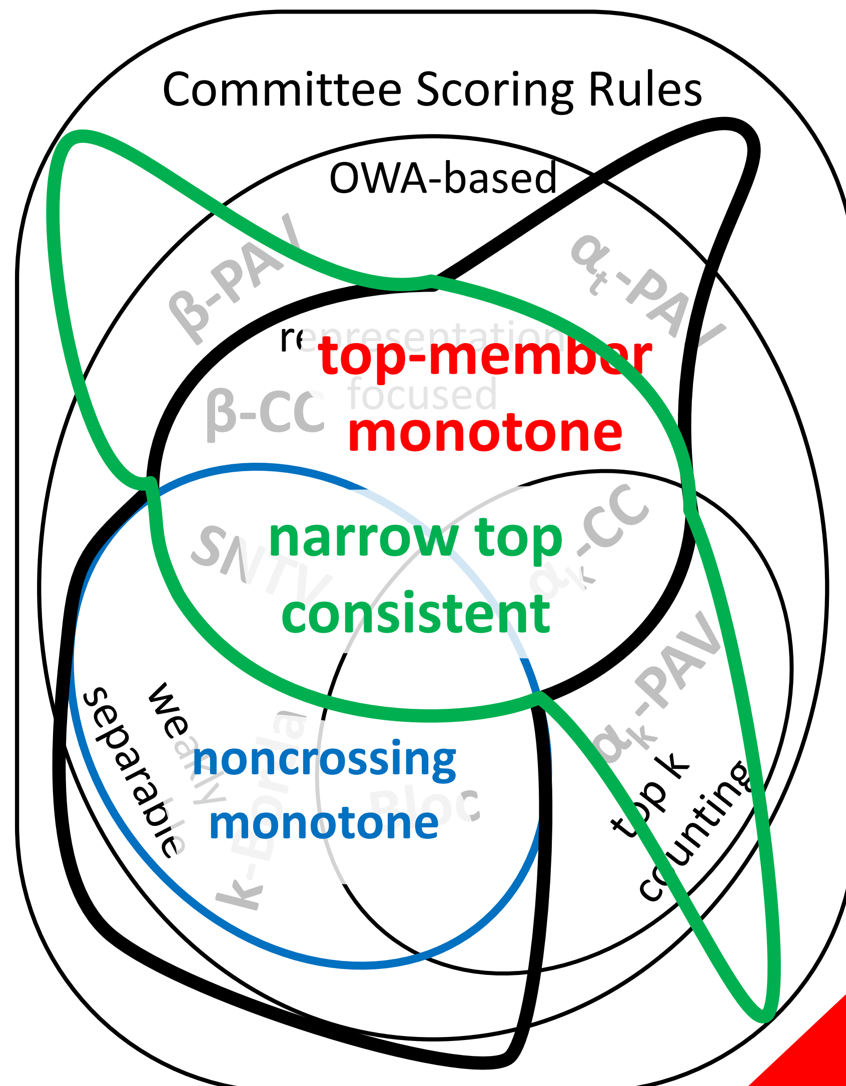
Axioms: Narrow Top + Top Member Monotonicity

Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

Top-Member Monotonicity: If the highest ranked member of the winning committee is moved forward, the committee still wins.

Narrow Top

A rule satisfies the **narrow top** criterion if whenever there is a set *W* of *k* candidates such that each voter ranks first a member of W, then W is a winning committee



[FSST16] P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Committtee Scoring Rules: Axiomatic Classification and Hierarchy, AAAI-2016

Axioms: Narrow Top + Top Member Monotonicity

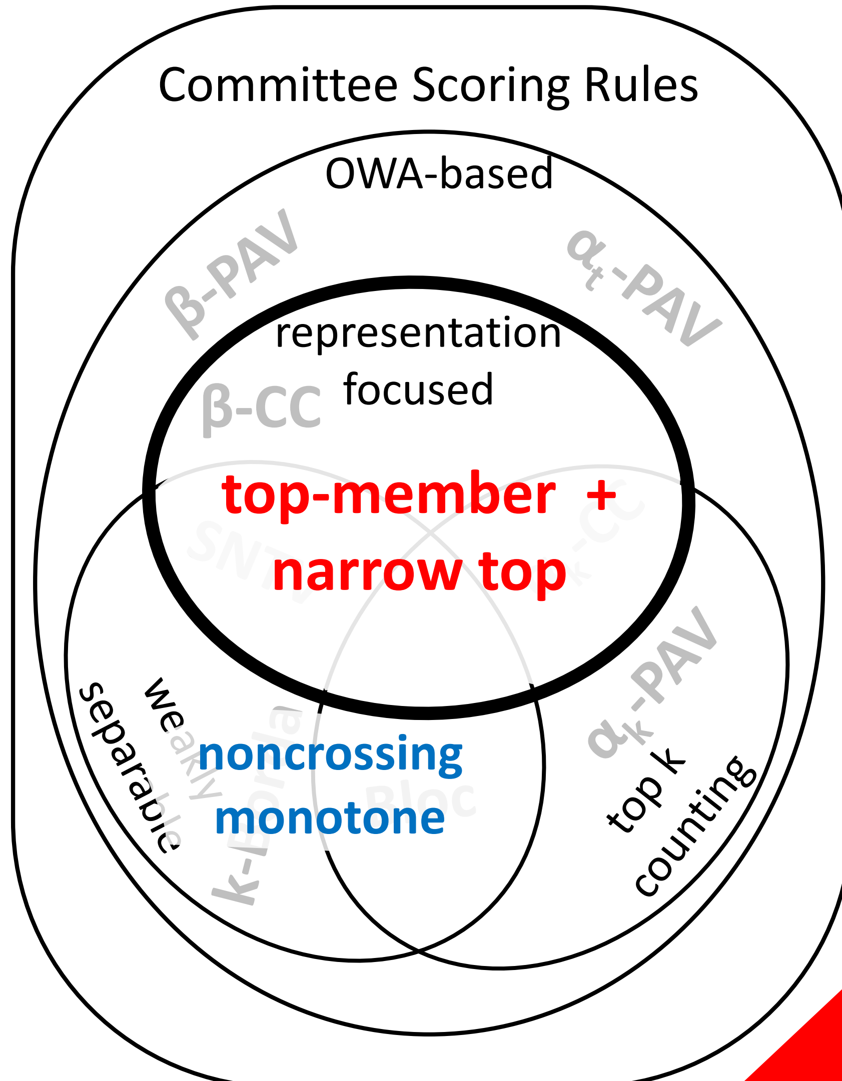
Noncrossing Monotonicity: If a member of the winning committee is moved forward (without passing another committee member), the committee is still winning

Top-Member Monotonicity: If the highest ranked member of the winning committee is moved forward, the committee still wins.

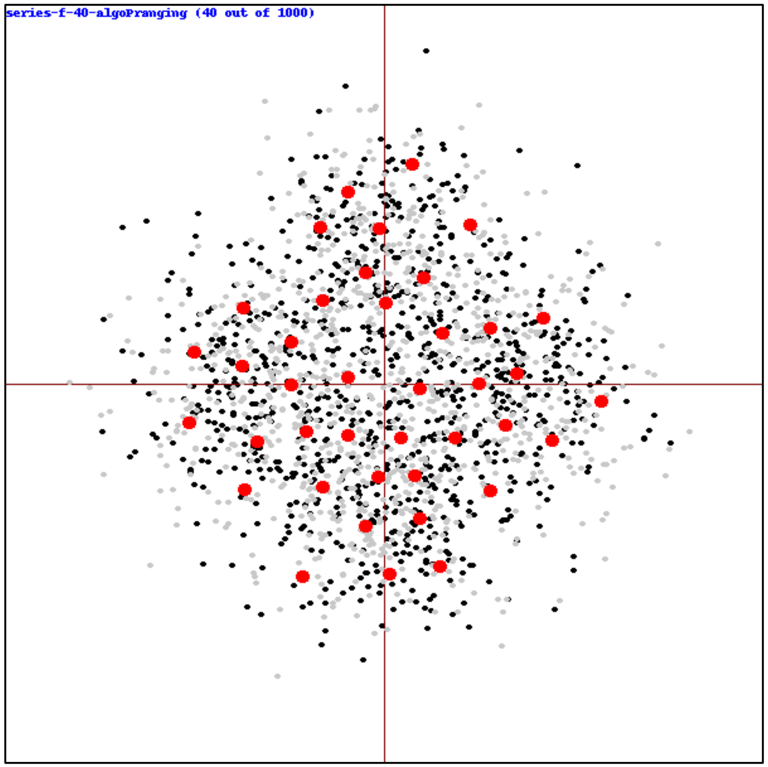
Narrow Top

A rule satisfies the **narrow top** criterion if whenever there is a set *W* of *k* candidates such that each voter ranks first a member of W, then W is a winning committee

Theorem A committee scoring rule is representation focused if and only if it is top-member monotone and consistent with the narrow-top principle.

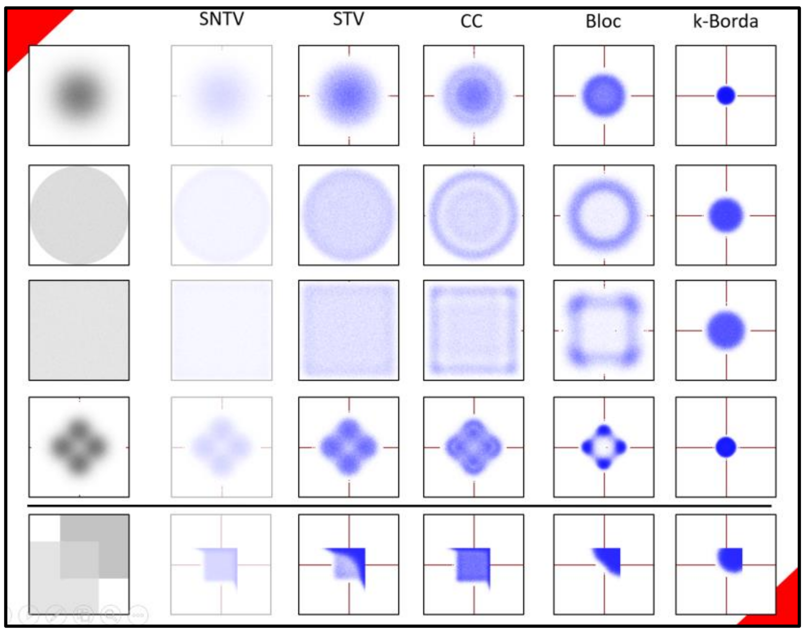


Chamberlin—Courant is good for diversity



Challenges

- **How to choose the right rules?**
  - How to decide if a rule is good?
  - How to design one?
  - How to compute committees?



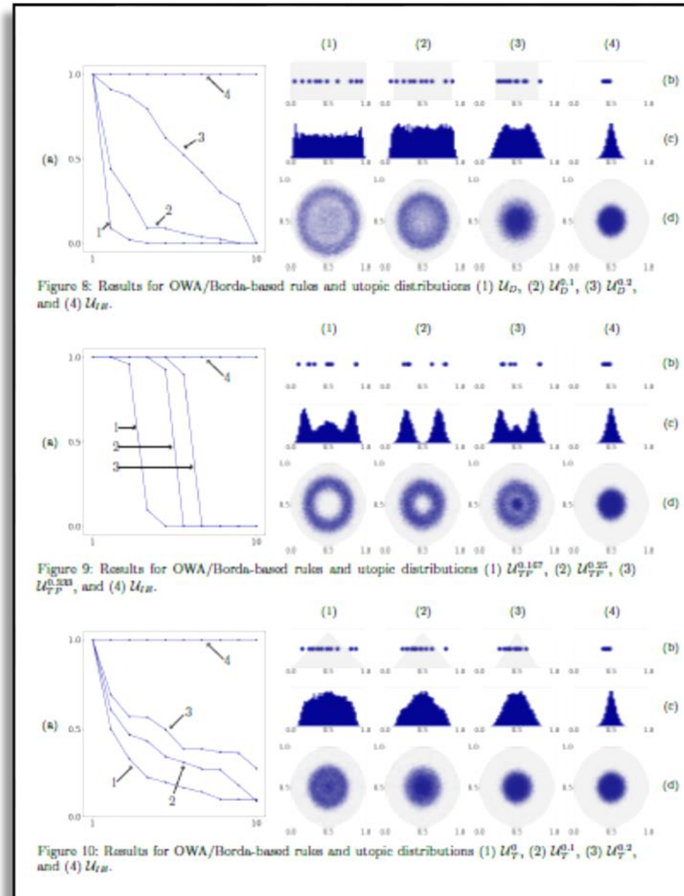


Figure 8: Results for OWA/Borda-based rules and utopic distributions (1) $\mathcal{U}_D$, (2) $\mathcal{U}_D^{0.1}$, (3) $\mathcal{U}_D^{0.2}$, and (4) $\mathcal{U}_{IR}$.

Figure 9: Results for OWA/Borda-based rules and utopic distributions (1) $\mathcal{U}_{TP}^{0.167}$, (2) $\mathcal{U}_{TP}^{0.25}$, (3) $\mathcal{U}_{TP}^{0.333}$, and (4) $\mathcal{U}_{IR}$.

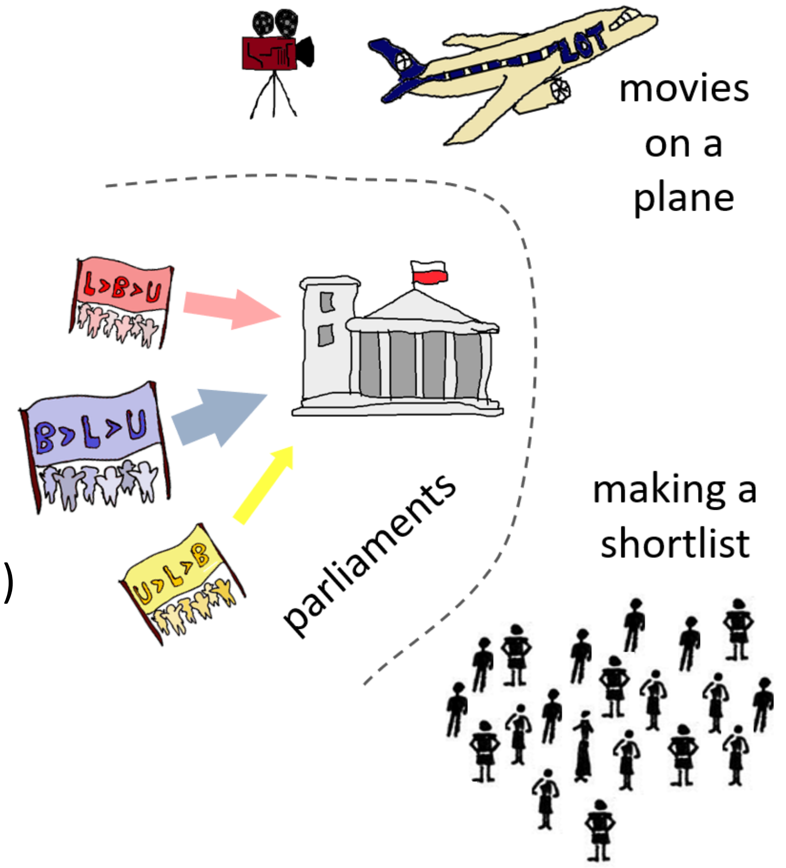
Figure 10: Results for OWA/Borda-based rules and utopic distributions (1) $\mathcal{U}_T^{0}$, (2) $\mathcal{U}_T^{0.1}$, (3) $\mathcal{U}_T^{0.2}$, and (4) $\mathcal{U}_{IR}$.

Challenges

- **How to choose the right rules?**
  - How to decide if a rule is good?
  - How to design one?
  - How to compute committees?

- **Practical applications?**
  - Participatory budgeting (getting there …)
  - Portfolio selection – possibly
  - Sports – yeah!
  - Politics? Nah…

- **How meaningful are current results?**
  - Game theory can help/spoil the results?
  - How people vote in reality?



Thank You!

https://github.com/elektronaj/MW2D

Multiwinner Voting: A New Challenge for Social Choice Theory, P. Faliszewski, P. Skowron, A. Slinko, N. Talmon, Trends in Computational Social Choice, 2017